



## Weak constraints in four-dimensional variational data assimilation

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The formulation of four-dimensional variational data assimilation (4D-Var) allows the incorporation of constraints into the cost function which need only be weakly satisfied. A common constraint imposed in this way is the requirement that the analysis be close to a previous short-range forecast or background field. In many environmental systems certain quantities are known to be conserved exactly or to be approximately conserved, but this knowledge is often not used directly within the assimilation system. In this study we investigate the benefit obtained from imposing these conservation properties as weak constraints in the 4D-Var problem.

To study this problem we make use of Kepler's two- and three-body systems for planetary orbits. These are Hamiltonian systems and, as such, are known to exactly conserve the Hamiltonian, which in this case is equal to the total energy. A 4D-Var scheme is designed in which two different constraints on the *a priori* information can be applied. The first is a standard background constraint, which controls the distance to a background state expressed in terms of the model variables. The second constraint uses the Hamiltonian calculated from the background state as a weak constraint on the Hamiltonian of the analysis. The formulation of this second constraint is shown to change the nature of the gradient equation, with components in different directions to the gradient obtained from the background constraint.

Using identical twin experiments we illustrate how the two constraints influence the final analysis in different ways. We show how by constraining the Hamiltonian quantity together with the background state it is possible to obtain a more accurate analysis than when using a background state alone. This is found to be true even if the numerical model respects the conservation properties of the system.