Dynamics of Nonlinear Resonances

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Traditionally wave turbulence is supposed to be composed of random, chaotic motions of interacting waves which are described statistically. The central object of the statistical wave turbulence theory (WTT) are non-resonant wave interactions; and its principal result is the wave kinetic equation for Kolmogorov-Zakharov energy spectra. Exact resonances and quasiresonances are explicitly excluded from WTT and have been supposed to be of importance only for large-scale wave motions (finite-size effects). The model of laminated wave turbulence (2006) demonstrates that small clusters of exact resonances also do appear in the inertial interval of wavenumbers, i.e. in mesoscopic regimes. Each cluster consists of a few connected triads or quartets (in three- and four-wave processes correspondingly). The dynamics of these clusters is therefore of utmost importance. Equations describing dynamics of a cluster may embody a generic mathematical model (2007) for studying various nonlinear phenomena, e.g. intraseasonal oscillations in the Earth atmosphere, drift wave instability in Tokamak plasma, zonal flows in oceans, freak waves, etc.

Qualitative study of cluster dynamics (2008) shows that energy flux within a cluster depends on the types of connection between each two neighboring triads or quartets (generalization of Hasselmann’s criterion of instability). For the most frequent occurrence of resonance clusters it is possible to construct additional integrals of motion (2009), which allows in some cases to integrate the corresponding dynamical system in quadratures. In other cases a set of physically relevant parameters can be determined, which yield integrable clusters. The study of Poincaré sections by numerical simulations with non-integrable clusters of resonances for random initial conditions shows that they are "close" to integrable.