



## A posteriori error covariances in variational data assimilation

V. Shutyaev (1), F.-X. Le Dimet (2), and I. Gejadze (3)

(1) Institute of Numerical Mathematics, Russian Academy of Sciences, Moscow, Russian Federation (shutyaev@inm.ras.ru),  
(2) MOISE project (CNRS, INRIA, UJF, INPG); LJK, Joseph Fourier University, Grenoble, France (ledimet@imag.fr), (3)  
Department of Civil Engineering, University of Strathclyde, Glasgow, UK (igor.gejadze@strath.ac.uk)

The problem of variational data assimilation for a nonlinear evolution model is formulated as an optimal control problem to find unknown model parameters such as initial and/or boundary conditions, right-hand sides (forcing), and distributed coefficients. A necessary optimality condition reduces the problem to the optimality system (see, e.g.[1]) which includes input errors (background and observation errors); hence the error in the optimal solution. The error in the optimal solution can be derived through the errors in the input data using the Hessian of the cost functional of an auxiliary data assimilation problem. For a deterministic case it was done in [2]. In [3], a similar result was obtained for the continuous operator formulation, where a nonlinear evolution problem with an unknown initial condition was considered, with random errors in the input data subjected to the normal distribution. Here we present an extension of the results reported in [3] for the case of other model parameters (boundary conditions, coefficients, etc.) and show that in a nonlinear case the a posteriori covariance operator of the optimal solution error can be approximated by the inverse Hessian of the auxiliary data assimilation problem based on the tangent linear model constraints. We also demonstrate that this approximation could be sufficiently accurate even though the tangent linear hypotheses is not valid. Numerical examples are presented for a nonlinear convection-diffusion problem.

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### References:

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