Instabilities in the thin disk’s dynamo problem

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It is believed, that variety of the magnetic fields observed in astrophysics and technics can be explained in terms of the dynamo theory. The main idea is that kinetic energy of the convective motions is transformed into the energy of the magnetic field. Magnetic field generation is the threshold phenomenon: it starts when magnetic Reynolds number $R_m$ reaches its critical value $R_m^{cr}$. After that magnetic field grows exponentially up to the moment, when it already can feed back on the flow. This influence does not come to the simple suppression of the motions and reducing $R_m$, rather to the change of the spectra of the fields closely connected to constraints caused by conservation of the magnetic energy and helicity [1]. Even after quenching the saturated velocity field is still large enough, so that $R_m \gg R_m^{cr}$.

It appears, that velocity field taken from the nonlinear problem (when the exponential growth of the magnetic field stopped) can still generate exponentially growing magnetic field providing that feedback of the magnetic field on the flow is omitted (kinematic dynamo regime) [7,8,2,6]. In other words, the problem of stability of the full dynamo equations including induction equation, the Navier-Stokes equation with the Lorentz force differs from the stability problem of the single induction equation with the given saturated velocity field taken from the full dynamo solution: stability of the first problem does not provide stability of the second one.

Here we consider this effect on an example of the one-dimensional dynamo model [3,5] in the thin disk (or shell) in more details. These equations can be analysed not only numerically but in some cases using asymptotic analysis as well. Following [4], we show analytically that for the steady solutions this kind of instabilities is absent. This regime corresponds to the galactic dynamo.

Non-steady, oscillatory regimes, typical for dynamo in the thin shell, have more complex behaviour. For the simple integral forms of $\alpha$-quenching, like $\alpha = \frac{\alpha_0}{1 + \frac{E_m}{E_m^{cr}}}$, where $E_m$ is an averaged over the volume magnetic energy, instability is absent. The similar result is for the case, when $\alpha$ is time averaged and has constant value. Such behaviour let us to conclude that we have deal with parametric resonance, where $\alpha$ oscillates with doubled frequency $\gamma = 2\omega$, and $\omega$ is a frequency of the nonlinear solution. We show, that in our model for the different boundary conditions only combination of parametric resonance and local quenching can produce such kind of instabilities.

References