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Arnold's structural theorem for the thick disk model describes the structure of planetary systems and circumstellar disks

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Ideal hydrodynamics and MHD are widely used in astrophysical models due to not only low viscosity but because of large scale of the systems. These general models allow to describe not only flows of neutral gas and magnetized plasma but evolution of many-body system as well. Our basic idea is to describe cosmic structures by 3D steady Euler equation of ideal fluid flow.

According to Arnold's structural theorem, 3D steady Euler equation of ideal incompressible fluid flow has solution with toroidal topology described by divergent free vector field. The minimal energy among all the fields with closed flow lines on 3D sphere has a structure called Hopf field. Stereographic projection of Hopf field into Euclidian space has image as field where flow lines are Villarceau circles lying on tori corresponding to the levels of Bernoulli function. Arnold's structural theorem is also applicable to steady ideal incompressible MHD and steady isentropic or isothermal flow of ideal compressible fluid. We tested Arnold's structural theorem in the 3D hydrodynamic model of thick disk formation. We have shown that optimal level of Bernoulli function corresponding to "optimal torus" exists, and relative surface free energy is minimal at this level. Corresponding inclination of Villarceau circles to accretion plane is $\pi/4$. Beat of oscillations with wave numbers corresponding to structural radii of "optimal torus" lying in accretion plane leads to scaling of "optimal tori" with factor:

$$K = 1 + \sin(\pi/4) \approx 1.7071... \, .$$

Set of scaled optimal tori form thick disk with divergence angle to accretion plane $\pi/4$. Considering optimal tori as precursors of planetary orbits, we found coincidence of scaling factors in Dermort's Law for semi-major axis of Solar system $K = 1.7002 \pm 0.018$ and HR8799 system $K = 1.68 \pm 0.05$ with the optimal one. Also, we analyzed the probability distribution of ratios of semi-major axes of neighbor planets in the known multiple-exoplanet systems. We found existence of two maxima. First maximum was centered at scaling factor $K = 1.67 \pm 0.017$ similar to optimal one and second one was centered at $K = 3.10 \pm 0.04$ similar to squared optimal one. Scaling factors of planetesimal belts of inner ring of β-Pictoris system, in three ring system HD113766, and binary ring systems HD141569, HD98800B, Vega also precisely correspond to the optimal one.

Then,we have analyzed the scaling factors of satellite systems. We have shown that due perturbation connected with coincidence of flow lines involved in toroidal flow around protoplanets and proto-Sun there was a deviations of the scaling factor depending on central body axial tilt and local ratio of semi-major axes to neighbor internal planet. Values for all the satellites systems - Galilean, Saturnian, Uranian, Neptunian, Martian, Plutonians and Haumea's - were calculated with good agreement with actual orbital parameters.

Within the model, we have analyzed dependence of the planets volumes on the semi-major axis. It is demonstrated that all regular satellites of giant planets with increasing volumes at increasing orbital semi-major axes have distribution of logarithms of relative volumes vs the logarithms of relative semi-major axes approximated with linear dependence and high degree of correlation. In suggestion on absence of dissipation of particles involved in toroidal flow around planets, high correlation confirms that density of particles in all satellite systems was similar. Moon has close parameters to concerned distribution, thereby suggesting that its formation has been processed by a common co-accretion mechanism. We found that Triton's parameters fit well the dependence of logarithm of relative volumes on logarithm of relative semi-major axes of regular Neptune's satellites. This means that formation of Triton and regular Neptune's satellites was simultaneous. We have shown that orbit inclination of Triton corresponded to its formation in opposite phase to regular satellites being similar to Plutonians and Haumea's systems.