Nonparametric estimation of seismic activity and seismic hazard

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To characterize seismic regime in an area it is important to determine seismic activity. It also can be used for seismic hazard estimation. It was suggested a new general method of seismic activity estimation for a certain point in space, time and magnitude. This method is based on nonparametric statistics methods.

Let's determine seismic activity \( A \) in a magnitude-spatial-temporal interval – \( (M, \Delta M, X, \Delta X, Y, \Delta Y, Z, \Delta Z, T, \Delta T) \) as number of seismic events \( N \) in this interval normalized to the size of this interval:

\[
A = \frac{N}{(\Delta X \Delta Y \Delta Z) \Delta T \Delta M} = \frac{N}{\Delta V \Delta T \Delta M}.
\]

It means that \( A = A(M, \Delta M, X, \Delta X, Y, \Delta Y, Z, \Delta Z, T, \Delta T) \) is a function of 10th parameters. Our goal will be estimation of seismic activity in a point, which is not depends on \( \Delta V, \Delta T \) and \( \Delta M \).

Let's considered interval will be a subinterval of a big interval \( I_{tot} \) with a large number of seismic events \( N_{tot} \). Then according to Bernoulli theorem ratio \( \frac{N}{N_{tot}} \) will be good probability estimation \( P \) for occurrence of next event in considered interval. In the same time the probability for small considered interval (when \( \Delta V, \Delta T, \Delta M \) are small) can be estimated as:

\[
P = p \Delta V \Delta T \Delta M,
\]

where \( p = p(M, X, Y, Z, T) \) is probability density function for occurrence of seismic event with magnitude \( M \) in time \( T \) and in point \( (X, Y, Z) \).

As a result seismic activity can be estimated as

\[
A = \frac{N}{\Delta V \Delta T \Delta M} \approx \frac{N_{tot}P}{\Delta V \Delta T \Delta M} \approx N_{tot}p = N_{tot}p(M, X, Y, Z, T).
\]

To estimate probability density function for occurrence of seismic event with magnitude \( M \), in time \( T \) and in point \( (X, Y, Z) \) the methods of nonparametric statistics can be used. Let's use Parzen-Rozenblatt estimation. In general for \( m \) variables \( x_1, x_2, ..., x_m \) it is

\[
\hat{p}_{N_{tot}}(x_1, x_2, ..., x_m) = \frac{1}{N_{tot}} \sum_{i=1}^{N_{tot}} \frac{1}{h_1 h_2 \cdot ... \cdot h_m} K \left( \frac{x_1 - x_{1i}}{h_1}, \frac{x_2 - x_{2i}}{h_2}, ..., \frac{x_m - x_{mi}}{h_m} \right),
\]

where \( K \) is a kernel function, \( h_1, h_2, ..., h_m \) are scaling parameters and \( (x_{1i}, x_{2i}, ..., x_{mi}) \) are coordinates of \( i \)-th experimental point.

There are some limitation for kernel function:

\[
\int K(\xi) d\xi = 1, \quad \int |K(\xi)| d\xi < \infty, \quad \sup |K(\xi)| < \infty, \quad \lim_{\xi \rightarrow \infty} |\xi K(\xi)| = 0,
\]

where \( \xi \) is multivariate value \( (\xi_1, \xi_2, ..., \xi_m) \).

For good qualities of \( p \) estimation, the scaling parameters \( h_j \) must depends on \( N_{tot} \) and answer following requirements:

\[
\lim_{N_{tot} \rightarrow \infty} N_{tot} h_1(N_{tot}) h_2(N_{tot}) \cdot ... \cdot h_m(N_{tot}) = \infty, \quad \lim_{N_{tot} \rightarrow \infty} \max_j (h_j(N_{tot})) = 0,
\]
\[
\lim_{N_{tot} \to \infty} N_{tot}[h_1(N_{tot})h_2(N_{tot}) \cdots h_m(N_{tot})]^2 = \infty.
\]

Parameters \( h_j = c_j(N_{tot})^{k/m} \), where \( 0 < k < 1/2 \) and \( 0 < c_j < \infty \) are suitable for this purposes.

As an example let use \( \exp(-|\xi|/h)/(2h) \) as a kernel function. In this case probability density function for occurrence of seismic event with magnitude \( M \), in time \( T \) and in point \((X, Y, Z)\) can be estimated as

\[
\hat{p}_{N_{tot}}(x, y, t, M) = \lim_{\Delta x, \Delta y, \Delta t, \Delta M \to 0} \frac{\Delta P(x, y, t, M, \Delta x, \Delta y, \Delta t, \Delta M)}{\Delta x \Delta y \Delta t \Delta K} =
\]

\[
= \frac{1}{N_{tot}} \sum_{i=1}^{N_{tot}} \left[ \frac{1}{16\pi h_{xy}^2 h_t h_M} \exp \left( -\sqrt{\left( \frac{x - x_i}{h_{xy}} \right)^2 + \left( \frac{y - y_i}{h_{xy}} \right)^2} \right) \exp \left( -\frac{|t - t_i|}{h_t} \right) \exp \left( -\frac{|M - M_i|}{h_M} \right) \right].
\]

This estimation of probability density function for occurrence of seismic event with magnitude \( M \), in time \( T \) and in point \((X, Y, Z)\) can be considered as a seismic hazard estimation. Seismic activity can be estimated as \( A(x, y, t, M) \approx N_{tot} \hat{p}_{N_{tot}}(x, y, t, M) \).

The results of the described approach application for investigation of seismic activity in Altay-Sayan region (Russia) are discussed.