Geophysical Research Abstracts
Vol. 13, EGU2011-1749, 2011
EGU General Assembly 2011

Parameter estimation of nonlinear dynamical systems using synchronization

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It is very important for climate science to model nonlinear dynamical systems, because geosystem is governed by highly nonlinear equation of motion with huge degrees of freedom. Even if we assume a model, it is a difficult task to estimate the parameters in the model. In this study, we propose a new method which estimates parameter values of nonlinear dynamical equations from measured data. We assume that the data x obeys an equation of motion

$$\dot{x} = \sum_{i=1}^{N} c_i f_i(x), \tag{1}$$

where x is a vector and f_i's are vector functions. Our goal is to estimate unknown parameters c_i. Here we employ the concept of synchronization between data and model; we run the numerical simulation of a model equation which contains the coupling term in order to achieve synchronization between them. We can expect that a good model has a good synchronizability to the data, although it is impossible to completely synchronize the model with data because we do not know the correct value of c_i. However, we found that if we introduce the nonlinear coupling between them in an appropriate way, synchronization error is proportional to the difference between the correct parameter value and one used in the model. This means that the information of synchronization error can be converted to the information of the parameter value of the system. If there are many unknown parameters ($N > 1$), we can decompose the synchronization error into the ones originated from $f_i(x)$ term. This decomposition enables us to estimate all of those unknown parameter values c_i. In the absence of noise, we prove that we can exactly achieve them by using the proposed method. Our method is applicable if we have no a priori knowledge about the system. In such a case we assume that the governing equation can be expanded in Taylor series and determine all coefficients with our method. Note that our method works even when the system shows the complex behavior, e.g., turbulent or chaotic motion.

Moreover, our method works even in the presence of noise. We numerically verified that parameters can be well estimated if there is additive noise term in the governing equation and in the measurement, if the noise is not very strong. We will study the dependence of the accuracy of the estimation on the noise intensity.

In many cases, it is impossible to measure all variables. Therefore we need to develop a method to achieve a good estimation with a few observables. We propose a method to apply the embedding technique which is widely used in nonlinear dynamics; we estimate the parameters for the evolution equation in the embedded space.

In the presentation, we describe the method and show that we can estimate all parameters exactly in an ideal case that all variables can be measured and there is no noise. We discuss detailed properties of the method such as the robustness of the method, extension to spatially dependent systems with large degrees of freedom, and cost for computation.