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Comparing attractor frequency bands with eigen frequencies for a confined rotating fluid

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Due to its simplicity, trapezoidal cross section was extensively use for studying wave attractors mainly for gravity waves [1,2]. For inertial waves a trapezoidal prism tank filled with a homogenous liquid (water) was investigated [3]. Due to the fact that inertial waves are essentially 3D, systems with rotational symmetry are adequate for the study of the inertial waves. The rotating spherical shell case [4] however, gives quite complicate attractor solutions. In order to combine the rotational symmetry and the trapezoidal cross section, the first geometry that occurs to us is a system consisting on a liquid confined between an outer vertical cylinder and a co-rotating coaxial inner cone. In the axial direction the liquid is bounded by horizontal plates. The cylinder and the cone are subject to the same average angular velocity. The inertial waves are forced by modulating the rotation rate of the inner cone. At small viscosities (or high angular velocity) inertial waves show multiple reflections at the walls before the waves are damped by diffusion.

From the mathematical point of view, the description of the waves that collapse to an attractor is given by a hyperbolic PDE. Generally one deals with ill-posed boundary value problems in the meaning that hyperbolic partial differential equations are combined with elliptic boundary conditions. However, for such problems one can find eigenmode solutions only for very limited number of domains. One of these domains is obtained for non-inclined walls of the inner cone. When the inner cylinder wall is inclined, the hyperbolicity leads to internal shear layers corresponding to singularities for the inviscid case (attractor solutions). The geometrical structure of the shear layers can be explained by inertial waves, trapped on limit cycles. The shape of the limit cycle defines the structure of the internal shear layers [1,5].

In fact, the spectrum of regular modes, existing for the case of vertical cylinder walls, vanishes when the inner wall is inclined. We will show that the 'spectrum' of limit cycles for weakly inclined walls still captures some part of the eigenvalue spectrum of trajectories for the vertical walls. To answer this question we compute the attractor frequency 'spectrum' for a small cylinder inclination angle and we compare it with the eigen spectrum for the case of vertical cylinder walls. If m is the number of reflection points of the attractor solution on the radial axis and n the number of the reflection points on the vertical axis one can observe that only (m-even, n-odd) and (m-odd, n-odd) attractors are obtained. It appears that the odd-even solutions are the eigenmodes surviving from the vertical wall problem. The intervals where attractors can be found collapse to distinct single frequencies when the wall inclination goes to zero. These values are the same as the eigenmode frequencies for small curvature (when cylinder radius is much larger than the cylindrical gap).

References

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