



Sparse-grid-based global sensitivity analysis for subsurface flow problems

A. Guadagnini (1), F. Nobile (2), M. Riva (1), L. Tamellini (2,3)

(1) DIIAR, Politecnico di Milano, Milano, Italy, (2) CSQI - MATHICSE, EPFL, Lausanne, Switzerland, (3) MOX - department of Mathematics, Politecnico di Milano, Milano, Italy

Consider a problem whose state variables \mathbf{u} are described by a set of equations depending on N parameters \mathbf{y} . In the case in which these parameters are not known exactly and are treated as random variables it is of interest to assess the influence of each of them on the state variables, i.e. to perform a global sensitivity analysis of the function $\mathbf{u} = \mathbf{u}(\mathbf{y})$.

Sobol' indices provide an effective way to tackle this problem. However, they are defined as high-dimensional integrals, and their computation with numerical quadrature schemes (e.g. Monte Carlo sampling) can be very expensive. A possible approach to ease the computational burden is to compute the Sobol' indices starting from a polynomial approximation of $\mathbf{u}(\mathbf{y})$. We consider the polynomial chaos expansion of $\mathbf{u}(\mathbf{y})$, which consists in projecting $\mathbf{u}(\mathbf{y})$ over a set of orthogonal polynomials in \mathbf{y} , and makes the computation of the Sobol' indices particularly easy.

Classically, the coefficients of the polynomial chaos expansion are computed either by projecting the equation with a Galerkin technique or by high-dimensional quadrature. Yet, the former approach may be infeasible for complex problems, and the latter may lead to inaccurate estimates. To this end, in this talk we propose a strategy that consists in computing a second type of polynomial approximation for $\mathbf{u}(\mathbf{y})$ (the so-called sparse grid approximation of $\mathbf{u}(\mathbf{y})$) which is then converted into a polynomial chaos expansion. This procedure is appealing since it only entails solving a number of uncoupled problems (in the same way as Monte Carlo) but can be much more efficient. We will show its effectiveness on some test cases describing saturated flows in porous media.