



Evaluating the role of higher order nonlinearity in water of finite and shallow depth with a direct numerical simulation method of Euler equations

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In deep water, the dynamics of surface gravity waves is dominated by the instability of wave packets to side band perturbations. This mechanism, which is a nonlinear third order in wave steepness effect, can lead to a particularly strong focusing of wave energy, which in turn results in the formation of waves of very large amplitude also known as freak or rogue waves [1]. In finite water depth, however, the interaction between waves and the ocean floor induces a mean current. This subtracts energy from wave instability and causes it to cease for relative water depth $\frac{h}{\lambda}$, where k is the wavenumber and h the water depth [2]. Yet, this contradicts field observations of extreme waves such as the infamous 26-m “New Year” wave that have mainly been recorded in regions of relatively shallow water. In this respect, recent studies [3] seem to suggest that higher order nonlinearity in water of finite depth may sustain instability.

In order to assess the role of higher order nonlinearity in water of finite and shallow depth, here we use a Higher Order Spectral Method [4] to simulate the evolution of surface gravity waves according to the Euler equations of motion. This method is based on an expansion of the vertical velocity about the surface elevation under the assumption of weak nonlinearity and has a great advantage of allowing the activation or deactivation of different orders of nonlinearity. The model is constructed to deal with an arbitrary order of nonlinearity and water depths so that finite and shallow water regimes can be analyzed. Several wave configurations are considered with oblique and collinear with the primary waves disturbances and different water depths. The analysis confirms that nonlinearity higher than third order play a substantial role in the destabilization of a primary wave train and subsequent growth of side band perturbations.

References

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