



A statistical mechanics approach to computing rare transitions in multi-stable turbulent geophysical flows

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Many turbulent flows undergo sporadic random transitions, after long periods of apparent statistical stationarity. For instance, paths of the Kuroshio [1], the Earth's magnetic field reversal, atmospheric flows [2], MHD experiments [3], 2D turbulence experiments [4,5], 3D flows [6] show this kind of behavior. The understanding of this phenomena is extremely difficult due to the complexity, the large number of degrees of freedom, and the non-equilibrium nature of these turbulent flows. It is however a key issue for many geophysical problems.

A straightforward study of these transitions, through a direct numerical simulation of the governing equations, is nearly always impracticable. This is mainly a complexity problem, due to the large number of degrees of freedom involved for genuine turbulent flows, and the extremely long time between two transitions.

In this talk, we consider two-dimensional and geostrophic turbulent models, with stochastic forces. We consider regimes where two or more attractors coexist. As an alternative to direct numerical simulation, we propose a non-equilibrium statistical mechanics approach to the computation of this phenomenon. Our strategy is based on large deviation theory [7], derived from a path integral representation of the stochastic process. Among the trajectories connecting two non-equilibrium attractors, we determine the most probable one. Moreover, we also determine the transition rates, and in which cases this most probable trajectory is a typical one.

Interestingly, we prove that in the class of models we consider, a mechanism exists for diffusion over sets of connected attractors. For the type of stochastic forces that allows this diffusion, the transition between attractors is not a rare event. It is then very difficult to characterize the flow as bistable. However for another class of stochastic forces, this diffusion mechanism is prevented, and genuine bistability or multi-stability is observed.

We discuss how these results are probably connected to the long debated existence of multi-stability in the atmosphere and oceans.

References

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