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Statistical Mechanics of the Shallow Water and Primitive Equations

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Geophysical flows are highly turbulent, yet embody large-scale coherent structures, such as ocean rings, jets, and large-scale circulations. Understanding how these structures appear and predicting their shape are major theoretical challenges.
The statistical mechanics approach to geophysical flows is a powerful complement to more conventional theoretical and numerical methods [1]. In the inertial limit, it allows to describe, with only a few thermodynamical parameters, the long-time behavior of the largest scales of the flow. Recent studies in quasi-geostrophic models provide encouraging results: a model of the Great Red Spot of Jupiter [2], an explanation of the drift properties of ocean rings [3], the inertial structure of mid-basin eastward jets [3], bistability phenomena in complex turbulent flows [4], and so on.
Generalization to more comprehensive hydrodynamical models, which include gravity wave dynamics and allow for the possibility of energy transfer through wave motion, would be extremely interesting. Namely, both are essential in understanding the geophysical flow energy balance. However, due to difficulties in essential theoretical parts of the statistical mechanics approach, previous methods describing statistical equilibria were up to now limited to the use of quasi-geostrophic models.

The current study fills this gap. The new theory we propose describes geophysical phenomena using statistical mechanics applied to the shallow water model, and is easily generalizable to the primitive equations. Invariant measures of the shallow water model are built based on the Hamiltonian structure and the Liouville theorem.
In parallel with the development of the theory, we devised an algorithm based on the Creutz algorithm [5] (a generalization of Metropolis-Hastings algorithm) in order to sample microcanonical measures. Numerical simulations are compared with the theoretical predictions [6].
We apply these new tools in order to describe vortex solutions similar to the Zapiola anticyclone. Part of the initial energy forms large-scale coherent structures, whereas the residual is transferred by gravity waves through the energy cascade to the smallest scales. We discuss the statistical mechanics prediction of the ratio of energy transferred to the smallest scales to the initial energy. This approach is the first theoretical quantitative prediction for this energy transfer.

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