



The problem of the highest wave in a group: fully nonlinear simulations of water wave breathers vs weakly nonlinear theory

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Deep water waves often propagate in groups due to the Benjamin – Feir instability. The weakly nonlinear theory for weakly modulated unidirectional waves within the framework of the integrable nonlinear Schrödinger equation (NLS) predicts that for an initially uniform wave train independently of its amplitude the highest wave in the group may be up to 3 times the uniform wave amplitude, while the most modulationally unstable wave results in about 2.4 times wave enhancement. What are the highest waves in reality is not known, the very notion is not well defined. We examine the problem of the highest wave in a modulated wave train by direct numerical simulations of exact Euler equations. In this case strong nonlinearity (including wave breaking) and strong linear and nonlinear dispersion may become crucial. The main purpose of the study is to re-define the notion of the highest wave and to establish the correspondence between the weakly nonlinear analytic NLS theory and simulations of strongly nonlinear analogues of the Akhmediev breathers (periodic in space modulations of a uniform wave) within the framework of the primitive hydrodynamic equations. The initial conditions specifying initial small perturbation of a uniform wave are chosen in such a way that only one predefined unstable mode (one breather) is excited. The simulations are performed for the range of initial steepness $ka = 0.04 \dots 0.25$ and the modulation length of from 2 to 20 carrier wave lengths. The simulations are focused on the final stage of the Benjamin – Feir instability, when large waves emerge out of weakly modulated wave trains. It is shown that in the non-breaking regimes wave crests may be amplified more than 4 times. Maximum wave trough amplification is shown to be more than 3 times; though the maximal wave height amplification in all non-breaking cases did not exceed 3. The moderation of wave height amplification is due to the short length of emerging wave groups: the wave height differs considerably from the doubled wave amplitude (strong dispersion). In the fully nonlinear model the focusing time and the threshold of modulation growth are greater than in the weakly nonlinear limit. The role of wave breaking is twofold: on the one hand, the already intense waves cannot be amplified significantly as they quickly break; on the other hand, close to the wave breaking onset the amplification factors of weaker waves increase sharply, so that near-breaking and breaking waves seem to represent the most dangerous case of wave dynamics. The results provide an important element for developing of deterministic forecasting of rogue waves.