Calculation of periodic flows in a continuously stratified fluid

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Analytic theory of disturbances generated by an oscillating compact source in a viscous continuously stratified fluid was constructed. Exact solution of the internal waves generation problem was constructed taking into account diffusivity effects. This analysis is based on set of fundamental equations of incompressible flows. The linearized problem of periodic flows in a continuously stratified fluid, generated by an oscillating part of the inclined plane was solved by methods of singular perturbation theory. A rectangular or disc placed on a sloping plane and oscillating linearly in an arbitrary direction was selected as a source of disturbances. The solutions include regularly perturbed on dissipative component functions describing internal waves and a family of singularly perturbed functions. One of the functions from the singular components family has an analogue in a homogeneous fluid that is a periodic or Stokes’ flow. Its thickness is defined by a universal micro scale depending on kinematics viscosity coefficient and a buoyancy frequency with a factor depending on the wave slope. Other singular perturbed functions are specific for stratified flows. Their thickness are defined the diffusion coefficient, kinematic viscosity and additional factor depending on geometry of the problem.

Fields of fluid density, velocity, vorticity, pressure, energy density and flux as well as forces acting on the source are calculated for different types of the sources. It is shown that most effective source of waves is the bi-piston. Complete 3D problem is transformed in various limiting cases that are into 2D problem for source in stratified or homogeneous fluid and the Stokes problem for an oscillating infinite plane.

The case of the "critical" angle that is equality of the emitting surface and the wave cone slope angles needs in separate investigations. In this case, the number of singular component is saved. Patterns of velocity and density fields were constructed and analyzed by methods of computational mathematics. Singular components of the solution affect the flow pattern of the inhomogeneous stratified fluid, not only near the source of the waves, but at a large distance. Analytical calculations of the structure of wave beams are matched with laboratory experiments. Some deviations at large distances from the source are formed due to the contribution of background wave field associated with seiches in the laboratory tank. In number of the experiments vortices with closed contours were observed on some distances from the disk.

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