Seismic waves in a self-gravitating planet

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The elastic-gravitational equations describe the propagation of seismic waves including the effect of self-gravitation. We rigorously derive and analyze this system of partial differential equations and boundary conditions for a general, uniformly rotating, elastic, but aspherical, inhomogeneous, and anisotropic, fluid-solid earth model, under minimal assumptions concerning the smoothness of material parameters and geometry. For this purpose we first establish a consistent mathematical formulation of the low regularity planetary model within the framework of nonlinear continuum mechanics. Using calculus of variations in a Sobolev space setting, we then show how the weak form of the linearized elastic-gravitational equations directly arises from Hamilton’s principle of stationary action. Finally we prove existence and uniqueness of weak solutions by the method of energy estimates and discuss additional regularity properties.