Parallel Discontinuous Galerkin based Geostatistical Inversion of Steady-State Flow and Transport Processes in Groundwater

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The hydraulic conductivity field of a groundwater aquifer is unknown and difficult to measure. We can only estimate it through direct or indirect measurement data of dependent quantities such as the hydraulic head or the concentration of a tracer. Comparing the discrepancy between real-world with simulated measurement data leads to the solution of an underdetermined inverse problem. Considering the unknown as a random process, multivariate statistical models can be used to describe the spatial correlation of the conductivity field. Furthermore, the Bayesian approach can be taken for the regularization of the inverse problem. The forward problems to be solved within the inversion procedure consist of the stationary elliptic groundwater equation and the formally elliptic, but nearly hyperbolic advection-dominated transport equation for very small transversal dispersivities. Both equations are coupled via the Darcy flux. The solution of the transport equation, especially in a heterogeneous flow field, may exhibit sharp fronts causing major numerical difficulties in two respects: firstly, numerical over- and undershoots at internal layers and, secondly, the arising linear system may become more difficult to solve when working with higher resolutions. Based on the discontinuous Galerkin method (DG), we present a robust numerical discretization of the coupled equations that can cope with high Péclet numbers, resolve the sharp fronts well and still keep the numerical oscillations at a level that is acceptable for the inversion scheme. In addition, we have implemented a method to solve the arising linear system efficiently.