Comparison of three methods of solution to the inverse problem of groundwater hydrology for multiple pumping stimulation

Mauro Giudici (1,2,3), Davide Casabianca (4), and Alessandro Comunian (1)

(1) Universita degli Studi di Milano, Dipartimento di Scienze della Terra, Milano, Italy (mauro.giudici@unimi.it, alessandro.comunian@unimi.it), (2) CINFAI (Consorzio interuniversitario nazionale per la fisica delle atmosfere e delle idrosfere), Tolentino (MC), Italy, (3) IDPA-CNR (Istituto per la dinamica dei processi ambientali, Consiglio Nazionale delle Ricerche, Milano, Italy, (4) Università degli Studi di Milano, Dipartimento di Fisica, Milano, Italy

The basic classical inverse problem of groundwater hydrology aims at determining aquifer transmissivity ($T$) from measurements of hydraulic head ($h$), estimates or measures of source terms and with the least possible knowledge on hydraulic transmissivity. The theory of inverse problems shows that this is an example of ill-posed problem, for which non-uniqueness and instability (or at least ill-conditioning) might preclude the computation of a physically acceptable solution. One of the methods to reduce the problems with non-uniqueness, ill-conditioning and instability is a tomographic approach, i.e., the use of data corresponding to independent flow situations. The latter might correspond to different hydraulic stimulations of the aquifer, i.e., to different pumping schedules and flux rates. Three inverse methods have been analyzed and tested to profit from the use of multiple sets of data: the Differential System Method (DSM), the Comparison Model Method (CMM) and the Double Constraint Method (DCM). DSM and CMM need $h$ all over the domain and thus the first step for their application is the interpolation of measurements of $h$ at sparse points. Moreover, they also need the knowledge of the source terms (aquifer recharge, well pumping rates) all over the aquifer. DSM is intrinsically based on the use of multiple data sets, which permit to write a first-order partial differential equation for $T$, whereas CMM and DCM were originally proposed to invert a single data set and have been extended to work with multiple data sets in this work. CMM and DCM are based on Darcy’s law, which is used to update an initial guess of the $T$ field with formulas based on a comparison of different hydraulic gradients. In particular, the CMM algorithm corrects the $T$ estimate with ratio of the observed hydraulic gradient and that obtained with a comparison model which shares the same boundary conditions and source terms as the model to be calibrated, but a tentative $T$ field. On the other hand the DCM algorithm applies the ratio of the hydraulic gradients obtained for two different forward models, one with the same boundary conditions and source terms as the model to be calibrated and the other one with prescribed head at the positions where in- or out-flow is known and $h$ is measured. For DCM and CMM, multiple stimulation is used by updating the $T$ field separately for each data set and then combining the resulting updated fields with different possible statistics (arithmetic, geometric or harmonic mean, median, least change, etc.). The three algorithms are tested and their characteristics and results are compared with a field data set, which was provided by prof. Fritz Stauffer (ETH) and corresponding to a pumping test in a thin alluvial aquifer in northern Switzerland. Three data sets are available and correspond to the undisturbed state, to the flow field created by a single pumping well and to the situation created by an “hydraulic dipole”, i.e., an extraction and an injection wells. These data sets permit to test the three inverse methods and the different options which can be chosen for their use.