



## Probability density functions for use when calculating standardised drought indices

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Time series of drought indices like the standardised precipitation index (SPI) and standardised flow index (SFI) require a statistical probability density function to be fitted to the observed (generally monthly) precipitation and river flow data. Once fitted, the quantiles are transformed to a Normal distribution with mean = 0 and standard deviation = 1. These transformed data are the SPI/SFI, which are widely used in drought studies, including for drought monitoring and early warning applications.

Different distributions were fitted to rainfall and river flow data accumulated over 1, 3, 6 and 12 months for 121 catchments in the United Kingdom. These catchments represent a range of catchment characteristics in a mid-latitude climate. Both rainfall and river flow data have a lower bound at 0, as rains and flows cannot be negative. Their empirical distributions also tend to have positive skewness, and therefore the Gamma distribution has often been a natural and suitable choice for describing the data statistically. However, after transformation of the data to Normal distributions to obtain the SPIs and SFIs for the 121 catchments, the distributions are rejected in 11% and 19% of cases, respectively, by the Shapiro-Wilk test. Three-parameter distributions traditionally used in hydrological applications, such as the Pearson type 3 for rainfall and the Generalised Logistic and Generalised Extreme Value distributions for river flow, tend to make the transformed data fit better, with rejection rates of 5% or less. However, none of these three-parameter distributions have a lower bound at zero. This means that the lower tail of the fitted distribution may potentially go below zero, which would result in a lower limit to the calculated SPI and SFI values (as observations can never reach into this lower tail of the theoretical distribution).

The Tweedie distribution can overcome the problems found when using either the Gamma or the above three-parameter distributions. The Tweedie is a three-parameter distribution which includes the Gamma distribution as a special case. It is bounded below at zero and has enough flexibility to fit most behaviours observed in the data. It does not always outperform the three-parameter distributions, but the rejection rates are similar. In addition, for certain parameter values the Tweedie distribution has a positive mass at zero, which means that ephemeral streams and months with zero rainfall can be modelled. It holds potential for wider application in drought studies in other climates and types of catchment.