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Efficient stencil assembly in global geodynamic models

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In mantle circulation models the simulation domain is a thick spherical shell representing the earth's mantle. Typically, finite elements are the method of choice to account for the spherical geometry. The wide range of length scales involved in earth dynamics is a major challenge. Capturing localized features such as faulted plate boundaries requires local resolutions in the order of $\mathcal{O}(1)$km whereas the radius of earth mantle is in $\mathcal{O}(10^4)$km. That is, a globally refined mesh of this resolution needs $\mathcal{O}(10^{13})$ degrees of freedom (DOF). To realize such extreme resolutions, most codes utilize adaptive mesh refinement techniques. However, this class of methods leads to an increase in algorithmic complexity which in turn often results in significant computational overheads and causes additional communication requirements and memory traffic.

In our new mantle convection solver, based on the hierarchical hybrid grids (HHG) framework, we pursue a different approach. The domain is represented by a coarse unstructured mesh of tetrahedral elements that are treated as macro elements. The macro elements are subjected to a structured global refinement. Exploiting the resulting regular mesh structure of the refined grids HHG achieves not only excellent parallel scalability but also superior node performance. These two features together with a matrix free implementation lead to superior performance and the possibility to solve the systems even for extreme numbers of DOFs, reaching in fact a global resolution of 1km for the whole earth mantle.

One difficulty with this approach is that the resolution of the curved geometry may be insufficient, since it is only defined by the relatively coarse macro elements. This can be improved by implementing a projection of all the refined nodes from each of the macro elements onto the spherical geometry. However, if this is implemented in the conventional way, it increases the computational cost significantly and slows the solver down by a large factor.

Here we introduce a novel approach that permits us to almost maintain the excellent run-time of the original HHG framework for the projected geometry. Instead of an exact assembly of the finite element stencils, we approximate the stencil values with trivariate quadratic polynomials within each macro tetrahedron. The polynomial coefficients are determined in the setup phase by a least-squares fit and are stored for each macro element. The evaluation of the polynomials is performed with an efficient incremental algorithm based on a Taylor series expansion which reduces the cost for each polynomial evaluation to only two additional FLOPs. The implementation of this algorithmically optimal scheme is further optimized based on a systematic node level performance analysis and using SSE vectorization. We will present an analysis of both the accuracy achieved as well as the computational efficiency for the Laplace equation as well as for the Stokes system.