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A study of almost adiabatic convection in thick liquid cores of terrestrial planets

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Abstract

The convection heat transfer resulting from the flow between two rapidly rotating concentric spheres in almost adiabatic approximation is considered. The choice of physical parameters is based, first of all, on the possible applications to the hydrodynamics of the deep interiors of the Earth and planets and moons of Terrestrial group. We simplify the initial system of equation using asymptotic analysis in a fast rotation limit to a single second-order ODE for the pressure or vertical (z) component of the velocity. A numerical solution to the problem is then obtained for a range of values of the Prandtl number.

1. Introduction

The present work forms the continuation of the paper [1], in which the basic equations describing marginal stability of almost adiabatic thermal and compositional convection in the deep planetary interiors was obtained and partially investigated. All considered objects, planets and moons of Terrestrial type, are in the state of rapid rotation, which means that fluid motion in its deep convective interiors is characterized by the large Reynolds number, or, equivalently, by the small Ekman number. The presence of this small parameter allows us using the asymptotic analysis to reduce the initial problem for the system of partial differential equations to the simplified two-point boundary value problem for a single second-order ordinary differential equation.
Pioneering researches of thermal convection in rapidly rotating spherical layer was started with the local asymptotic analysis [2], [6] and corresponding solutions and critical parameters differ from numerical solution of initial system of the partial differential equations by an $O(1)$ amount. The part of our previous investigation of almost adiabatic planetary convection [9] was also based on the local theory which considered local onset of convection in a sphere with homogeneous distribution of heat sources. Also, in [9] some non-uniform distributions of the sources of thermal and/or compositional convection in a spherical layer, modeling deep planetary interiors, were investigated asymptotically and numerically for the limited set of Prandtl numbers. In this paper general solutions for the large and small Prandtl numbers Pr are received and the previous local solution for $\mathrm{Pr}=1$ is essentially corrected following global WKBJ approach [3], [4]. Obtained asymptotical critical Rayleigh numbers and critical frequencies are in a good agreement with direct numerical calculations for the initial system of PDEs [3]. However, even the most advanced numerical calculations are limited by the values of the Ekman numbers down to 10^{-7} which are much greater than the typical values for the deep interiors of planets and moons, while asymptotic solutions can operate with as small Ekman numbers as required. And another advantage of asymptotic solutions is possibility to consider a wider range of physical parameters than in direct numerical simulations.
Also in this paper critical Rayleigh numbers, frequencies and distributions of almost adiabatic planetary convection are received completely analytically for the range of possible Prandtl numbers in sufficiently thick spherical layers. The latter means that the inner radius of the spherical layer r_i is much smaller than the outer radius r_o. Then it is possible to consider their ration $b=r_i/r_o$ as additional small parameter in the system which allows us to use WKBJ approach and find analytical solution for the system of two ODEs for the vertical

velocity and the pressure. In this case we also assume that the heat source distribution is non-uniform with maximum of heat power on the inner boundary. For example, this distribution corresponds to the situation when the temperature gradient is maintained by the temperature difference between the inner and outer boundaries. Or, more naturally, when the heat transfers from the inner and outer boundaries are different and the power of the heat sources in the spherical volume is negligibly small.

2. Summary and Conclusions

Our results for the large Prandtl numbers can be interpreted within the frames of compositional or gravitational-chemical planetary convection theory [1]. The corresponding critical frequencies are almost steady-state that is in agreement with steady-state behavior of the Main geomagnetic field which, as it is generally accepted, is generated by the compositional convection. Thermal convection is much less effective for generation of a magnetic field. Therefore almost adiabatic thermal convection likely dominates in almost non-magnetic interiors of Venus, Mars, Moon and the majority of other large moons. Small and asymmetric (about the rotation axis) own magnetic field of Mercury and some moons of Jupiter are possibly generated by the more powerful thermal convection. These suggestions are in agreement with our results for the small Prandtl numbers which correspond to thermal convection and lead to high critical frequencies.

In the deep interiors of giant planets thermal convection is much more intensive than in the interiors of planets and moons of Terrestrial group. It follows from the astronomical data [5], theories [8] and direct satellite observations over heat fluxes and magnetic fields [5],[8]. Thus, in this case, thermal convection alone is able to generate powerful enough magnetic field [8]. Accordingly, we consider various distributions of convective sources at Prandtl number unity since it is typical value for the deep interiors of planets and moons with vigourous basic uniform convection.

The most interesting result is analytical expressions for critical Rayleigh numbers, frequencies and distributions of almost adiabatic planetary convection for all possible Prandtl numbers in the approximation of small radius aspect ratio $b = r_i / r_o$. We consider non-uniform distribution of convective sources with the maximum source power at the inner boundary which forms when the heat transfer rate differs on the inner and outer boundaries and power of heat sources in the volume of the spherical layer is negligibly small [9]. Similar distribution of thermal and/or compositional convective sources is likely typical for the deep convective interior of the most part of the planets and moons since it is possible to omit a homogeneous radioactive heating [7].

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