The Ill-Effects of Traditional Approximation in Exoplanet Atmospheric Flow Modeling

J. Y-K. Cho, I. Polichtchouk
Astronomy Unit, School of Mathematical Sciences, Queen Mary, University of London, London E1 4NS, UK
(j.cho@qmul.ac.uk, I.Polichtchouk@qmul.ac.uk)

Abstract
We investigate the effects of imposing the traditional approximation restriction in large-scale atmospheric flow models of exoplanets. In the traditional hydrostatic primitive equations, solved by general circulation and global climate models, terms associated with the horizontal components of the Coriolis acceleration and the metric terms are omitted. However, scale analysis suggests that the neglected terms may be up to \(\sim 20\%\) of the included terms in the traditional equations.

In this work, we enhance a well-tested general circulation model to include the omitted terms consistently. In direct comparisons of simulations appropriate for hot extrasolar giant planets, we demonstrate a significant difference when the terms normally neglected in the approximation are retained; for example, large warm/cool regions (of up to \(\sim 300\)K) persist and large-scale jets disappear. The effect is most prominent when rotation does not strongly suppress vertical motion, as in most close-in exoplanets. It is also prominent during the early stages of any planet simulation, in which the atmosphere/climate is spun up from rest.

1. Background
Most general circulation models (GCMs) of extrasolar planet atmospheres are based on the standard primitive equations [1]. In addition to the hydrostatic balance, these equations make the traditional approximation, in which the \("\cos \phi\) Coriolis" and “metric” terms from the full Navier-Stokes equations are concurrently omitted. For the Earth, the traditional approximation is valid to an acceptable degree—especially for adiabatic, balanced, synoptic-scale motions. However, the approximation is not as valid for other planets and for unbalanced flows. This can be particularly significant for exoplanets subject to strong diabatic forcing and/or unusual physical conditions (e.g., short-period eccentricity, large radius, etc.). It is also significant, in principle, for simulations of any planetary atmospheres “spun up” from rest. In this work, we use a three-dimensional, stably-stratified hydrodynamics model that self-consistently retains the terms omitted in the traditional approximation to investigate the effects of relaxing the approximation in exoplanet simulations.

We use the equations of [2]. Their “non-traditional” primitive equations do not make the traditional approximation but still retain energy, axial angular momentum, and potential vorticity conservation (as in the traditional primitive equations). Here we discuss simulations of three tidally-synchronized model hot giant planets with different rotation rates: except for the rotation rate, the physical parameters of the planet are all taken to be those for HD209458b (see, e.g., [3]). Net heating due to the host star is represented with simple Newtonian relaxation, characterized by the relaxation time and the equilibrium potential temperature. Identical sets of simulations are performed using the “non-traditional” and “traditional” equations (hereafter WBE and HPE, respectively). Integration is carried out in all simulations from rest up to 100 planetary rotations. The vertical domain is from 1 to 1000 mbar, resolved by up to 20 layers in the vertical direction and T85 spectral resolution in the horizontal direction.

2. Results
Scaling analysis shows that rotation rate \(\Omega\) strongly affects the importance of non-traditional effects, as do the characteristic scale height \(H\) and characteristic flow speed \(U\) (see, e.g., [2]). Specifically, the importance of keeping the \(\cos \phi\) Coriolis terms can be assessed by considering the following comparison: \(|2\Omega w \cos \phi|/|Du/Dt| \lesssim 2\Omega H \cos \phi/U\), where \(w\) is the vertical velocity and \(\phi\) is the latitude. If this condition is true, \(\cos \phi\)—as well as the metric—terms should be included. For the Earth, the differences be-
tween the WBE and HPE are small, except when di-
abatically forced, synoptic-scale motions in the trop-
ics and planetary-scale motions are dominant. For the
typical hot extrasolar giant planet (with HD209458b parame-
ters), the difference is also minor, when $U \approx 1000 \text{ m s}^{-1}$. However, when $U \approx 100 \text{ m s}^{-1}$, the dif-
fferences are not minor ($\lesssim 0.2$). For giant planets on
shorter periods than HD209458b, the difference can
be very significant ($\approx 0.7$). In fact, given the sensi-
tivity to the initial condition observed in climate sim-
ulations starting from the rest state (see, e.g., [3]), the
differences in all “spin-up” simulations of any planet
is likely to be significant.

Figure 1 shows the difference between temperature
and flow field of the direct simulation of HD209458b
with and without the traditionally neglected terms.
In the simulations, the location of vortices and hot
regions for WBE is different than for HPE. This is
because relaxing the traditional approximation intro-
duces, in addition to a rotationally-modified Rossby
mode, smaller scale perturbations. The small-scale
fluctuations in the flow obtained with WBE is gener-
ated by the vertical velocity terms absent in HPE.

The basic comparison presented above is repeated
with varying Rossby number $Ro$, which measures
the ratio of $U$ to $\Omega$ for a fixed characteristic length
scale (planet radius here). For large Rossby number
($Ro \approx 1.5$), the differences in the flow structure are
even more noticeable than for the moderate Rossby
number ($Ro \approx 0.5$) case, shown in Figure 1. For small
Rossby number ($Ro \approx 0.1$) case, the differences in
the flow structure are less than in the moderate and large
$Ro$ cases. The main difference in the small $Ro$ case
is that the WBE simulation does not exhibit a strong
westward polar jet, as in the HPE simulation. Both
WBE and HPE simulations show a more zonal flow,
compared with the moderate and large $Ro$ cases—as
expected for small $Ro$ flows in general.

3. Conclusion

The traditional approximation in general circulation
models of hot exoplanets leads to results which are
significantly different, compared with results obtained
with models not making the approximation. This
presents another cautionary tale for judicious applica-
tion of models, theoretical and numerical, originally
developed for Solar System planets (Earth in partic-
ular): equations with full Coriolis force and metric
terms should be used to obtain more robust results for
exoplanets. We have used HD209458b as a paradigm
hot extrasolar planet to investigate the significance of
neglecting certain terms in the hydrodynamics equa-
tions. However, the consequences can also be signif-
icant for any planet which possesses flows with fast
vertical and/or slow horizontal motions. Future work
will investigate these effects more carefully when the
small scales are better represented with higher resolu-
tion.

Acknowledgements

J.Y-K.C. was supported by the Science and Technol-
ogy Facilities Council grant PP/E001858/1, and I.P.
was supported by a research studentship from the
Westfield Trust.

References

(Academic Press, San Diego), 1992

Quasi-hydrostatic Equations for Global Models with a
Complete Representation of the Coriolis Force, Q. J.

tial Flow on Close-in Planet Atmospheric Circulation’,