

Effects of asteroids on the orbital motions of terrestrial planets

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Abstract

This work is presented in order to try to assess with as much detail as possible the effects of the largest asteroids on the orbits of terrestrial planets Mercury, Venus, Earth and Mars. The present planetary ephemerides, as INPOP08 (Fienga et al 2009) and DE405 (Standish et al, 1998) are subject to a lack of accuracy, because of the perturbations arising from a large number of asteroids. Those perturbations could reach a few kilometers in several decades in the case of Mars for instance. So it looks appropriate to model in details the individual specific effects of these asteroids. For that our methodology consisted of several stages: a numerical integration of the orbits of the planets with and without the disturbing asteroid from which we want to know the effects; a determination of the signal representing the effects, by simple subtraction; an analysis of this signal by FFT (Fast Fourier Transform / Fast Fourier Transform); and adjustment of the signal by the set of sinusoids determined in the previous step.

This type of study is interesting in many fields, such as planetary ephemerides, as well as spatial navigation, to understand better the effects of each asteroid taken individually on the terrestrial planets. Note that this type of study is a continuation of previous studies (Williams 1984; Mouret et al 2009)

1. Introduction

The motion of a given planet around the Sun can be considered at first approximation as a Keplerian motion perturbed by the other planets and the small bodies of the solar system. Each of these perturbations must be treated either analytically or numerically, and can be measured as a change of the planet's osculating orbital elements ($a, e, i, \Omega, \varpi = \Omega + w$ and $L = \varpi + M$) determined from the perturbing function $\mathcal{R}$, according to Lagrange's formula

$$\begin{aligned} \frac{da}{dt} &= \frac{2}{na} \frac{\partial \mathcal{R}}{\partial L} \\ \frac{de}{dt} &= -\frac{\sqrt{1-e^2}}{na^2 e} \left(1 - \sqrt{1-e^2}\right) \frac{\partial \mathcal{R}}{\partial L} - \frac{\sqrt{1-e^2}}{na^2 e} \frac{\partial \mathcal{R}}{\partial \varpi} \\ \frac{di}{dt} &= -\frac{1}{na^2 \sqrt{1-e^2} \sin i} \left[\frac{\partial \mathcal{R}}{\partial \Omega} + (1 - \cos i) \left(\frac{\partial \mathcal{R}}{\partial \varpi} + \frac{\partial \mathcal{R}}{\partial L} \right) \right] \\ \frac{d\Omega}{dt} &= \frac{1}{na^2 \sqrt{1-e^2} \sin i} \frac{\partial \mathcal{R}}{\partial i} \\ \frac{d\varpi}{dt} &= \frac{\sqrt{1-e^2}}{na^2 e} \frac{\partial \mathcal{R}}{\partial e} + \frac{1 - \cos i}{na^2 \sqrt{1-e^2} \sin i} \frac{\partial \mathcal{R}}{\partial i} \\ \frac{dL}{dt} &= n - \frac{2}{na} \frac{\partial \mathcal{R}}{\partial a} + \sqrt{1-e^2} \left(\frac{1 - \sqrt{1-e^2}}{na^2 e} \right) \frac{\partial \mathcal{R}}{\partial e} + \\ &\quad + \frac{1 - \cos i}{na^2 \sqrt{1-e^2} \sin i} \frac{\partial \mathcal{R}}{\partial i} \end{aligned}$$

2. Effects on Mars

To evaluate the effects of a given asteroid on the terrestrial planets we use the numerical integration (Runge-Kutta of the 12th order), in the frame of the 9-body problem (the Sun and the eight planets without asteroids), then of the 10-body problem (the Sun and the eight planets together with the given asteroid). Then we determine the differential variations of orbital parameters of the planet by simple subtraction of the two signals obtained. After we perform the frequency analysis of the data, using fast Fourier transform (FFT) to determine the leading frequencies. At last we carry out a nonlinear regression in which the differential data are modeled by least-square method following an equation of type :

$$F(t) = \sum_{i=1}^N A_i \sin(f_i t) + B_i \cos(f_i t) + C_i t \sin(f_i t) + D_i t \cos(f_i t)$$

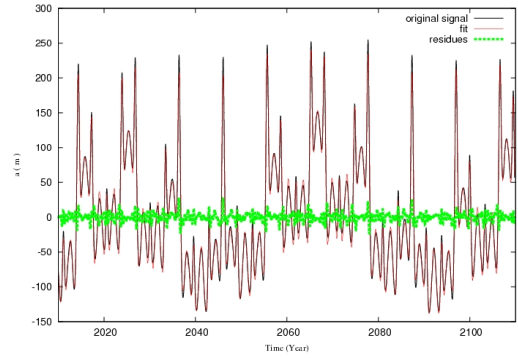
We present below an example of our results: in the next Table we show the maximum effects of each asteroid on the orbital parameters of Mars, for the periods of 100 years.

asteroide	Δa [m]	$\Delta e \times a_{Mars}$	Δi ["] $\times 10^{-6}$	$\Delta \Omega$ ["] $\times 10^{-6}$	$\Delta \varpi$ ["] $\times 10^{-6}$	$\Delta \lambda$ ["] $\times 10^{-6}$
1 Ceres	385.99973	1018.25341	837.97915	37190.92552	16391.84563	19419.20264
2 Pallas	193.24460	413.14399	441.88830	12173.16362	3700.64524	18125.33458
3 Juno	29.79749	47.96241	35.77520	869.55154	430.40624	3198.86497
4 Vesta	404.55975	1636.34551	586.32212	8301.55218	14585.61346	64574.80064
6 Hebe	61.28165	319.84333	74.70642	1275.36428	2971.28411	4854.97680
7 Iris	55.05084	116.68998	9.41114	838.18920	1050.72348	1612.14343
8 Flora	22.17044	65.30294	27.75361	131.25152	362.96686	2024.49401
9 Metis	66.42932	218.60586	11.70010	876.90488	2098.95546	2956.91830
10 Hygiea	40.41443	244.13825	38.31212	1254.45965	2798.38321	2998.78852
11 Parthenope	7.79214	51.54678	7.89198	42.35986	505.50615	464.19872
13 Egeria	11.49283	23.51995	5.00169	1626.31059	361.99825	619.36587
14 Irene	12.16370	21.63927	13.84427	483.07604	271.56248	1050.89770
15 Eunomia	22.84708	71.25421	64.06555	1315.85795	470.31980	2029.37351
16 Psyche	6.99493	21.21170	9.41352	229.74675	161.67922	645.65713
17 Thetis	2.17981	7.78699	2.12143	5.41765	68.21111	219.80180
18 Melpomene	6.50754	13.38326	14.99738	180.46621	105.09344	119.62906
19 Fortuna	25.64017	117.05157	1.45028	241.74259	1004.49811	6000.85764
20 Massalia	43.15692	87.96440	0.52064	92.24314	785.42295	2598.67141
21 Lutetia	4.85095	36.77710	0.95140	13.72545	290.94054	488.24872
22 Kalliope	2.24835	8.23517	3.41134	335.01872	66.82759	193.40492
24 Themis	8.29007	35.55968	0.45115	40.77149	263.71602	2145.45218
28 Bellona	13.00662	27.20348	20.89924	241.43927	481.31076	666.97955
29 Amphitrite	14.75650	49.64686	21.15240	273.76387	582.36206	1189.20823
31 Euphrosyne	5.12241	22.00394	5.49823	240.62689	302.94482	359.52655
45 Eugenia	7.74860	23.94010	19.99005	233.45638	290.71744	267.90582
46 Hestia	17.96786	63.84851	10.11392	600.04503	585.88586	1087.43324
47 Aglaja	0.56693	1.52783	0.98627	13.06890	14.46588	3.34039
48 Doris	2.88513	13.51154	4.41259	192.22558	143.69507	293.43395
49 Pales	1.36624	6.58591	0.74581	33.42004	44.87418	46.59249
52 Europa	6.43481	26.91968	13.23229	69.31637	342.50298	1296.25556
65 Cybele	2.39575	7.49899	3.32215	76.06273	71.36757	325.08383
87 Sylvia	1.31168	3.38215	3.47084	213.27108	43.60731	26.55712
88 Thisea	4.08374	12.87875	6.41856	289.82627	119.82741	613.18320
90 Antiope	0.42065	2.01203	0.04924	0.74104	18.39564	53.38957
107 Camilla	1.45676	3.18390	5.05085	147.73203	63.08296	17.97039
111 Ate	144.29700	397.32342	171.68672	4215.37702	5169.29002	31583.25731
121 Hermione	0.65030	1.99023	0.94898	52.20717	20.46184	138.98339
130 Elektra	2.82965	10.37911	10.78122	110.47445	115.07394	132.25069
165 Loricel	6.50281	27.76875	28.27952	465.04322	345.34894	231.09589
189 Phthia	0.06564	0.31375	0.02352	2.13209	3.13691	8.40880
243 Ida	0.02475	0.03559	0.00574	0.31220	0.46578	2.44193
253 Mathilde	0.11630	0.19786	0.13028	4.48961	2.69104	8.82521
283 Emma	0.35806	1.11882	1.04708	16.94125	9.49715	9.82103

The following Table shows the coefficients of Fourier and Poisson components for the effects of Ceres on the semi-major axis of Mars.

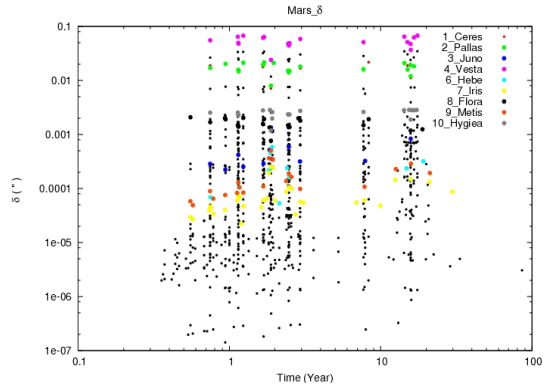
BIAS : -0.781486×10^{-5}									
LINEAR : 0.834730×10^{-2}									
T^2 : -0.221019×10^{-1}									
PERIOD (Day)	PERIOD (Year)	SIN $\times 10^4$	COS $\times 10^4$	T SIN $\times 10^0$	T COS $\times 10^0$	amplitud $\times 10^3$	Asteroid		
37126.67	101.65	1.5936	-0.1788	1.6036	-7.7484	0.7782	1.1005	7.0983	1_Ceres
17079.57	46.76	0.0805	0.0941	0.1238	-0.3470	-0.4554	0.6441	6.4984	1_Ceres
4849.40	13.28	-0.0024	0.0022	0.0032	0.0144	-0.0085	0.0120	5.0240	1_Ceres
3752.00	10.27	0.0038	0.0056	0.0068	0.0076	-0.0154	0.0217	3.6213	1_Ceres
3050.05	8.35	-0.0018	0.0000	0.0018	0.0087	-0.0077	0.0109	2.6364	1_Ceres
1876.35	5.14	0.0015	0.0029	0.0033	0.0040	-0.0178	0.0251	2.0555	1_Ceres
1250.96	3.42	0.0009	-0.0003	0.0009	-0.0028	-0.0013	0.0018	1.8923	1_Ceres
1161.21	3.18	-0.0003	-0.0025	0.0025	-0.0017	0.0058	0.0081	1.9559	1_Ceres
887.00	2.43	0.0001	-0.0022	0.0022	-0.0041	-0.0064	0.0090	1.5842	1_Ceres
717.51	1.96	0.0003	0.0034	0.0034	-0.0033	-0.0214	0.0302	1.1753	1_Ceres
580.74	1.59	-0.0011	-0.0004	0.0012	-0.0137	0.0163	0.0230	0.9827	1_Ceres
502.92	1.38	-0.0011	-0.0001	0.0011	0.0027	0.0089	0.0126	0.8156	1_Ceres
443.48	1.21	-0.0003	0.0011	0.0011	0.0042	-0.0031	0.0044	1.0453	1_Ceres
387.15	1.06	-0.0009	0.0018	0.0020	0.0136	-0.0116	0.0164	0.7292	1_Ceres
350.96	0.96	0.0019	0.0001	0.0019	-0.0058	-0.0037	0.0052	0.6818	1_Ceres
320.96	0.88	-0.0001	0.0004	0.0004	0.0009	-0.0039	0.0055	2.2882	1_Ceres
314.74	0.86	-0.0011	0.0010	0.0015	0.0045	-0.0028	0.0039	0.4754	1_Ceres
290.36	0.79	0.0011	-0.0005	0.0012	-0.0091	0.0021	0.0030	0.6036	1_Ceres
269.60	0.74	0.0005	0.0002	0.0006	-0.0027	0.0016	0.0022	0.4074	1_Ceres
DISPERSION		Before= 80.665229		After= 6.568432					
AMPLITUDE		Before= 390.753521		After= 49.948845					

The corresponding curves is preseted in the next Figure (the initial signal in black, the adjustment determined by our analysis in red and the residuals in green). In each case: (planet, astroid, orbital element) we find that our fit is satisfactory.



We have also calculated the individual influences of each asteroid on the distance from the EMB to the given planet and the orientation vector of this planet as seen from the EMB ,which are very important parameters in space navigation and astrometry.

The next Figure presents the amplitude with respect to each frequency found from our analyses of the individual influence of asteroids on the distance EMB-Mars. There are very clear vertical lines, related to the synodic period of the Mars.



References

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