Radiative heat transfer
in Saturn’s B ring

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Abstract
A new model of heat transfer by radiation has been elaborated to study the diurnal and seasonal temperature variations of Saturn’s rings as measured by the infrared CIRS-CASSINI spectrometer. The radiative heat transfer is treated with the heat diffusion equation, disentangling ring and particle thermal inertias. The ring thermal inertia is here a complex function of the ring and particles properties.

1. Introduction
The sun and the planet heat up planetary rings. The temperature of the lit face changes with the obliquity of sunrays on the ring plane along planetary seasons. Depending on the vertical structure and particles properties, the temperature of the unlit face will follow these variations. Although visible sunlight may not be directly transmitted in thickest rings, heat can reach the unlit face, as it has observed in the thickest B ring of Saturn [1]. This heat may be transported either by the vertical motion of ring particles or by radiation through successive absorption and emission, eventually scattering, or by conduction through contacts between particles or within the particles.

Most radiative transfer models developed to study the ring thermal emission are relying on the classical approach, assuming zero volume-filling factor D, i.e. very large inter-particle distances, or thickness-to-particle-size ratio H/R>>1. They also do not take into account shadow-hiding between particles. Morishima et al [3] recently introduced in a similar model the heat transport by vertical motion of particles and radiation, including also transient regime around the planetary shadow. Few thermal models have been developed to deal with rings of non-zero volume filling factor, typically monolayers [2]. If the largest particles are almost certainly spread in a monolayer, smaller ones may be distributed in a multilayer.

A new model is presented here that treats heat transport by radiation and conduction whatever the vertical structure. Assuming no vertical motion within the rings, it models the thermal gradient between lit and unlit faces as a function of solar elevation, transients in the planetary shadow, assuming non-zero volume filling factor D, thickness H, inter-particle shadow-hiding between particles covered with a porous regolith.

2. The model
2.1 Radiative heat diffusion
Classical equations of radiative transfer in stellar interiors can be combined to yield a diffusion equation with a radiative conductivity $K_r$ proportional to $T^3$ where $T$ is the local temperature in an isotropic radiation field. In particulate media, like packed beds in the extreme case, heat transport may happen by radiation through voids and conduction through contacts and solid phase of particles. The effective conductivity of the medium has thus several components. The radiative term can be written as $K_r=8RF_\tau\sigma T^3$, where $F_\tau$ is the radiation exchange factor, R the particle size, $\sigma$ the Stefan constant. The most difficult step is to determine the dependence of $F_\tau$ with the vertical structure of the layer, i.e. the optical depth $\tau$, the filling factor D, the relative thickness H/R, the particles solid conductivity $K_s$, their porosity $p$ or emissivity $\varepsilon$ [4]. The heat conduction through the solid phase of particles can also be included in the radiative conductivity $K_r$ as an impacting parameter [4]. Highly conducting spheres improve the exchange factor by favoring heat transfer through the solid phase whereas poorly conducting or porous ones will limit it. A porous ring with low D will also favor the exchange. The heat transport is governed by the diffusion equation

$$K_r \frac{\partial T}{\partial z} = \rho C_p \frac{\partial T}{\partial t}$$

(1)

where $C_p$ is the heat capacity of the ring layer and $\rho$ its volume density. The heat diffusion is solved with
a Crank-Nicholson algorithm with radiative boundary conditions.

2.2 Ring and particles physical properties

In this model, the ring thermal inertia \( \Gamma_r \), which controls the ring cooling in the planetary shadow is different from the particles thermal inertia \( \Gamma_p \): 

\[
\Gamma_r(T) = \sqrt{8RF_r\sigma T^3D(1 - p)\rho_0C_p} \quad \text{with} \quad D = \frac{4}{3}\frac{R}{H}
\]

where \( R \) is the particles size and \( \rho_0 \) the volume density of solid water ice. It also controls the vertical heat diffusion over the timescale \( t_v = H^2/\alpha_r \), where \( \alpha_r = K_r/\Gamma_r \) is the ring diffusivity. It depends on multiple factors and is no more a clear echo of the particle regolith properties as previously assumed ([2], [3]).

3. Application to the B ring

Temperatures of the lit and unlit faces, their variations around the planetary shadow and with increasing solar elevation \( B_0 \), are the main observables to constrain the ring and particle properties from infrared observations. The B ring is the densest ring of Saturn, with an optical depth up to 6 and a thickness that might of a few meters only. The vertical motion of particles is questioned. The formalism developed above is therefore applicable. It has been recently shown that the temperature of the unlit face increases with solar elevation despite a total opacity to visible light. The heat flux hit the unlit face in less than a dozen days [1].

Diurnal and seasonal variations of \( T_{UL} \) and \( T_L \) have been calculated versus ring and particle physical properties. \( F_E \) and \( D \) are the main factors at controlling the vertical diffusion and the thermal gradient between faces (Figure 1). The non-shadowed fractional area \( C(\tau, \mu) \) also controls the seasonal variations. The ring and particles porosities \( 1-D \) and \( p \) highly impact the ring thermal inertia. The resulting ring thermal inertia fixes the width of temperature variations on the lit face due to shadow crossing at each orbit, which appear as the dispersion of the red curve as shown in Figure 1.

4. Summary and Conclusions

This model calculates the heat transfer with the diffusion equation in dense rings with no vertical particle motions. Ring and particle thermal inertias are disentangled. Diurnal and seasonal temperatures variations can be predicted. Comparison of the model observables with CIRS data will help understanding the vertical structure and particle thermal properties of Saturn’s rings.

Figure 1: Temperatures of the lit (red) and unlit (blue) faces vs solar elevation \( B_0 \) for a ring with \( \tau=3.26 \), \( D=0.04 \) and \( H=10 \) m. The particles Bond albedo is 0.5, \( p=0.97 \) and \( \epsilon=0.9 \). The exchange factor is \( F_E=30 \). The resulting thermal inertia \( \Gamma \) ranges between 7 and 17 J/m²/K/s1/2 for particles inertia of 64 J/m²/K/s1/2.

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References


