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Multifractal model for Phobos crater size-frequency distribution

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Abstract

We present analytical functions which well reproduce the essential features of empirical crater size-frequency distributions for the Martian satellite Phobos. The functions are derived using reasonable simple assumption, that the crater size-frequency distributions (CSFDs) for different areas of Phobos surface have a multifractal character. We also obtained the model curves of CSFDs for the heavily cratered areas on the anti-Mars and sub-Mars hemispheres of Phobos.

1. Introduction

Impact craters of different sizes and levels of degradation are the dominant landforms on Phobos. The size-frequency distribution of the craters can help us to answer the following questions: What are the rates of formation and destruction of Phobos craters? Which craters have formed earlier, and which later? What is the age of Phobos? How is the size-frequency distribution of Phobos craters related with the mass distribution of the meteorites bombarding its surface? Therefore, it is very important to find the form of CSFDs for Phobos.

2. Model

In this study we present a multifractal model of CSFD for an area of Phobos surface. We will assume that CSFD has a multifractal nature. Then the following relation is true [1]:

$$p_i = N^{-\alpha_i} \text{ if } N \to \infty , \qquad (1)$$

where $p_i = (d_i - d_{\min}) / D_N$ are normalized diameters of craters forming a sample of size N ; d_i is the diameter of the i -th crater in the sample, $i = 1,...,N$; $d_{\min}$ is the diameter of the smallest crater in the sample; $D_N = \sum_{i=1}^{N} d_i - N d_{\min}$; α_i demonstrate how fast p_i decrease with the growth of N , $i = 1,...,N$.

The relative number of craters in the sample, characterized by the same value of α , with increasing the sample size N grows by a power law:

$$P(\alpha) = \frac{N_\alpha}{N} \propto N^{f(\alpha)-1} , \qquad (2)$$

where $f(\alpha)$ is the fractal dimension of a subset of craters with same α .

The spectrum $f(\alpha)$ can be approximated near its maximum by a parabola determined by the multifractal parameter α_0 :

$$f(\alpha) = 1 - \frac{(\alpha - \alpha_0)^2}{4(\alpha_0 - D_0)} . \qquad (3)$$

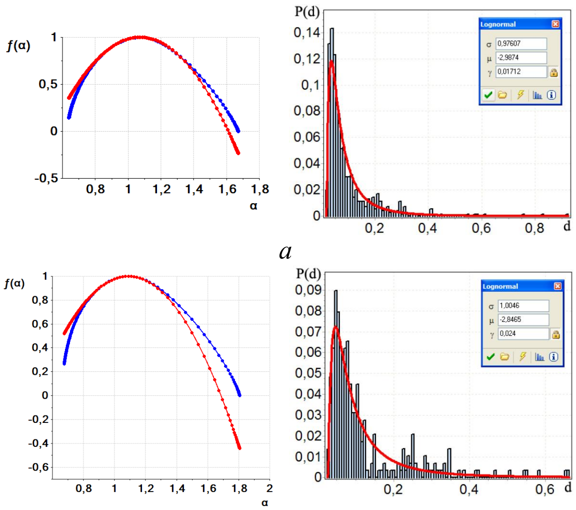
Substituting (3) in (2) and using that $\alpha = -\ln p \ / \ln N$ and $p = (d - d_{\min}) / D_N$, we obtain the lognormal distribution of crater diameters d , which can be written in the following form:

$$P[\ln(d - \gamma)] \propto \exp\left\{ -\frac{[\ln(d - \gamma) - \mu]^2}{2\sigma^2} \right\} , \qquad (4)$$

where $\mu = \ln D_N - \alpha_0 \ln N$, $\sigma^2 = 2(\alpha_0 - 1) \ln N$ and $\gamma = d_{\min}$.

3. Results

We obtained model curves for the size-frequency distributions of small craters (with diameter less than 1 km) of heavily cratered areas on the anti-Mars and sub-Mars hemispheres of Phobos. To obtain the model curves, we used the craters from a global catalog [2]. Boundaries of craters from this catalog were identified in Phobos images obtained by HRSC camera on Mars Express. The obtained multifractal spectra $f(\alpha)$ and their parabolic approximations for each of the Phobos areas are shown in Fig. 1. As seen from this figure, the spectra $f(\alpha)$ (blue curves) and their parabolic approximations (red curves) coincide near the maximum of $f(\alpha)$, and therefore, CSFDs for the heavily cratered areas on the anti-Mars and sub-Mars hemispheres satisfy a lognormal law. Fig. 1 also shows the lognormal curves for the distributions of crater diameters (in km), obtained through spectra $f(\alpha)$ for each of our areas on both hemispheres.



b

Figure 1: Multifractal spectra and CSFDs, obtained for the heavily cratered areas on the anti-Mars (*a*) and sub-Mars (*b*) hemispheres of Phobos.

For verification of our multifractal models of CSFDs we used the Kolmogorov-Smirnov and Anderson-Darling goodness of fit tests and measured (using EasyFit software, http://www.mathwave.com/) the "distance" between the sample data and model curves of CSFDs. As can be seen in Fig. 2, for each of our areas the distance (called the test statistic) is less than the threshold value (the critical value) calculated for the significance level (alpha) of 0.01. Therefore, our hypothesis, that the data follow the lognormal distribution, can be accepted.

As seen in Fig. 1, on the sub-Mars hemisphere, craters in diameter 45 m dominate, while on the anti-Mars side craters in diameter 35 m dominate. Moreover, on both hemispheres, a deficit of small craters with diameter less than 30 m is observed, probably due to limits in Mars Express image resolution and lighting conditions.

Kolmogorov-Smirnov		
Sample Size Statistic P-Value Rank	700 0,05717 0,01979 17	
α	0,02	0,01
Critical Value	0,05738	0,06157
Reject?	No	No
Anderson-Darling		
Sample Size Statistic Rank	700 3,4534 13	
α	0,02	0,01
Critical Value	3,2892	3,9074
Reject?	Yes	No

a

Kolmogorov-Smirnov		
Sample Size Statistic P-Value Rank	290 0,06182 0,2089 5	
α	0,02	0,01
Critical Value	0,08914	0,09566
Reject?	No	No
Anderson-Darling		
Sample Size Statistic Rank	290 1,7304 12	
α	0,02	0,01
Critical Value	3,2892	3,9074
Reject?	No	No

b

Figure 2: EasyFit interactive reports allowing to evaluate the goodness of fit for our lognormal models of CSFDs for the heavily cratered areas on the anti-Mars (*a*) and sub-Mars (*b*) hemispheres of Phobos.

4. Summary and Conclusions

In our study we established, that CSFDs for the heavily cratered (resurfaced) anti-Mars and sub-Mars hemispheres of Phobos satisfy a lognormal law. Our model curves of Phobos CSFDs are derived using the hypothesis, that CSFDs for different areas of Phobos surface have a multifractal character. In future, we plan to use model curves of CSFDs to estimate the ages of the anti-Mars and sub-Mars sides of Phobos.

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