Influence of the tides on the obliquity of Enceladus

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Abstract

We propose a Cassini State model for Enceladus in which we assume the presence of a global ocean beneath an ice shell and include the existence of deformations induced by obliquity tides. We aim to assess whether the obliquity of Enceladus can reach values large enough to trigger a strong heat flux caused by the dissipation associated with obliquity tides.

1. Introduction

Enceladus presents plumes of water vapor and ice particles at its warm south pole, which have been interpreted as evidence of a liquid water reservoir beneath the surface [6]. Gravity, topography, and heat flux measurements are in favor of a local reservoir. However, the gravity measurements cannot rule out a global ocean [5]. Spin precession measurements and modeling may help to discriminate between the two hypotheses, since a global ocean may decouple the surface from the interior and lead to different spin precessions of the solid layers.

For a synchronous triaxial satellite, it is often assumed that the obliquity is that of the classical Cassini state, valid for uniform precession. Here, we propose to develop a new theoretical model for the Cassini State, taking into account that Enceladus experiences a nonuniform orbital precession (e.g. [3] for the Galilean satellites), may harbor an internal global ocean (e.g. [1] for Titan), and experiences periodic elastic deformations. The need for a new model comes from the hypothesis that the jets of Enceladus could be associated with a strong heat flux resulting from tidal heating, if the obliquity is larger than 0.05 – 0.1° [7, 8]. In our modeling, we search for a resonant amplification of the solution (due to the nonuniformity of the orbital precession and/or the presence of the internal global ocean and/or the elastic deformations of the solid layers). Such a resonance may lead to a high values for obliquity, as in the case for Titan [1] or Ganymede and Callisto [2].

2. Solid spin precession

When the orbital precession is uniform and the satellite is solid and rigid, the obliquity \( \varepsilon \) of the Cassini state derived from the solution of the angular momentum equation governing the spin precession is constant

\[
\varepsilon = -\frac{i\dot{\Omega}}{\kappa + \Omega}, \quad \text{with} \quad \kappa = \frac{3}{2}n(C - A)
\]

where \( \dot{\Omega} \) is the constant orbital precession rate, \( i \) is the orbital inclination (constant if the precession is uniform), \( n \) is the mean motion, and \( A < B < C \) are the principal moments of inertia. Using the measured gravity field of [5], and the orbital precession of [9], we find that \( \varepsilon = 6.4 \times 10^{-4} \) degrees, or 2.80 m at the surface (see also [4]).

The effect of elastic deformations is to reduce the effective torque exerted by Saturn on Enceladus, in such a way that the “solid elastic obliquity” \( \varepsilon_{el} \) is given by Eq. (1) where \( \kappa \) is replaced by

\[
\kappa = \frac{3}{2}n \left\{ (C - A) - k_2 M_e R^2 q_r \right\}
\]

with \( k_2 \) the Tidal Love number of the solid Enceladus, \( M_e \) and \( R \) the mass and the mean radius, and \( q_r \) the ratio of the centrifugal acceleration to the gravitational acceleration. For \( k_2 = 0.015, \varepsilon_{el} \) is about 1% larger than \( \varepsilon \), and no resonant amplification occurs. Considering the nonuniform orbital precession of the satellite (see e.g. ephemerides of [9]) does not allow to find a resonant amplification neither (see Fig. 1).

3. Partly decoupled spin precession

We assume that Enceladus is made of three uniform and homogenous layers: an elastic ice shell (s), a liquid water ocean, and an elastic rock interior (i). Under the assumption that the ocean is in hydrostatic equilibrium, the system of angular momentum equations governing the spin precessions of the different layers...
reduces to the angular momentum equations of the solid layers [1]. They are written, correct up to the first order in spin obliquities and orbital inclination, as

\[ C_s \frac{dS_s}{dt} = I \tilde{\kappa}_s(N - S_s) - I \tilde{K}_s(S_i - S_s) \quad (3) \]

\[ C_i \frac{dS_i}{dt} = I \tilde{\kappa}_i(N - S_i) + I \tilde{K}_i(S_i - S_s) \quad (4) \]

where \( S_s \) and \( S_i \) are the projections on the Laplace plane of the unit vectors along the rotation axes of the shell and of the interior, respectively. \( C_s \) and \( C_i \) are the polar moment of inertia of the two solid layers. \( \tilde{\kappa}_s \) and \( \tilde{K}_i \) represent the strengths of the external torque of Saturn and of the internal gravitational torque on the solid layer \( l \), corrected for the ocean pressure and taking into account the elasticity effects. The solution of the system is characterized by an obliquity amplitude for the shell and the interior, for each term of frequency \( \dot{\Omega}_j \) and amplitude \( i_j \) of the series for the orbital precession:

\[ \varepsilon_{j,s} = \frac{i_j \dot{\Omega}_j(C_i \tilde{K}_i + C_s \tilde{\kappa}_s - C_s \tilde{\kappa}_i - C_i \tilde{K}_s) - C_i C_s (\omega_+ + \dot{\Omega}_j)(\omega_- + \dot{\Omega}_j)}{C_i C_s (\omega_+ + \dot{\Omega}_j)(\omega_- + \dot{\Omega}_j)}, \quad (5) \]

\[ \varepsilon_{j,i} = \frac{i_j \dot{\Omega}_j(C_i \tilde{K}_i + C_s \tilde{\kappa}_s - C_s \tilde{\kappa}_i - C_i \tilde{K}_s) - C_i C_s (\omega_+ + \dot{\Omega}_j)(\omega_- + \dot{\Omega}_j)}{C_i C_s (\omega_+ + \dot{\Omega}_j)(\omega_- + \dot{\Omega}_j)} \quad (6) \]

with \( \omega_{\pm} \), the eigenfrequencies of the system. We compute the solution for a large range of density profiles of Enceladus. We find that the obliquity of Enceladus’ shell is of the same order (\( \sim 6 \times 10^{-4} \) deg) as the obliquity of the solid case and that elasticity has an effect of at best a few percents on the solution.

### 4. Summary and Conclusions

We conclude that both the hypotheses of a local reservoir or of a global ocean fail to reach large obliquity values satisfying the requirements of [7, 8], even when nonuniform orbital precession and/or elastic tidal deformations are considered. We reach similar conclusions as previous study [4], that is to say that the origin of the strong heat flux of Enceladus is unlikely to be the dissipation associated with the obliquity tides.

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### References


