EPSC Abstracts
Vol. 10, EPSC2015-511, 2015
European Planetary Science Congress 2015

A tracers method for studying double diffusive convection in the liquid layers of planetary interiors

M. Bouffard (1,2), S. Labrosse (1), G. Choblet (2), A. Fournier (3), J. Aubert (3) and P. Tackley (4)
(1) ENS Lyon, France, (2) LPG, Nantes, France, (3) IPG, Paris, France, (4) ETH, Zurich, Switerzland
(mathieu.bouffard@ens-lyon.fr)

Abstract

Convection in the liquid layers of planetary interiors is usually driven by a combination of thermal and compositional sources of buoyancy. The low molecular diffusivity of composition causes troubles in the description of this field on the Eulerian grids typically employed in current codes of geodynamo because numerical diffusion on these grids is potentially larger than the real diffusivity. We developed a Lagrangian description of composition based on a method of tracers. The absence of numerical diffusion inherent to this method allows modeling of thermo-chemical convection with infinite Lewis number. The validation of this new tool on benchmark cases will be presented at EPSC as well as its first applications to the ocean of Ganymede with consistently coupled boundary conditions for temperature and composition.

1. Introduction

The liquid part of planetary cores is assumed to be composed of a mixture of iron and nickel, plus a small fraction of light elements, probably sulphur, oxygen or silicon [7]. Convection in these layers is usually driven by the combination of two sources of buoyancy: a thermal source directly related to the planet's secular cooling, the release of latent heat and the heat generated by radioactive decay, and a compositional source due to some process of cristallisation, for example the growth of a solid inner core which releases light elements into the liquid outer core. The molecular diffusivity of composition is at least 3 orders of magnitude lower than that of temperature [1], which can produce differences in the dynamics inherent to thermal and compositional convection, respectively.

The classical approach that has been proposed by Braginsky and Roberts [1] and widely adopted since consists in combining both sources of buoyancy into a single component named codensity, under the assumption that the action of turbulence simply enhances the diffusivities of both fields to a same turbulent value. Codensity is a very convenient approach but it remains limited and simplistic. Firstly, it does not allow for a correct description of the distinct and coupled boundary conditions for temperature and composition. Secondly, due to the complexities of core turbulence which is probably anisotropic ([1],[6]), codensity turns out to be only a rough approximation and its use may be particularly problematic inside stratified layers ([6],[5]), in which it is likely that turbulence will be much less efficient if not absent, causing the mixing of properties being rather performed by molecular diffusion [1]. For a more rigorous description, one should therefore solve distinct transport equations for temperature and composition using two different diffusivities. This "double diffusivity" scenario has already been studied ([6],[2],[8]) but only through a limited exploration of the parameters space, the ratio of the thermal and chemical diffusivities, called the Lewis number $Le = \kappa_T/\kappa_C$, being kept below a value of 10 in these studies. The reason for this limitation is purely technical and closely related to the fact that describing physical variables on a Eulerian grid generates numerical diffusion, this latter being potentially larger than the actual molecular diffusivities and therefore hiding appropriate transport phenomena with low diffusivities. In this work, we developed a Lagrangian description of composition based on the introduction of tracers in a Eulerian grid. Such a method theoretically guarantees the absence of numerical diffusion.

2. Principle of the method

A Lagrangian method is used to solve the transport equation for composition C, in which $\mathbf{u}$ is the velocity and Pr_C the compositional Prandtl number:

$$\frac{\partial C}{\partial t} + \mathbf{u} \cdot \nabla C = \frac{1}{Pr_C} \nabla^2 C \qquad (1)$$

A large number of particles (tracers) are initially dispersed through the Eulerian grid. Each tracer contains

its coordinates and the value of composition at its position. At each time step, the tracers are advected by the flow via a 4th-order Runge Kutta scheme by interpolating the velocity from grid nodes to their positions. This interpolation is quadratic in the direction along which there are strong variations of the velocity gradient and linear in the other directions. In the vicinity of poles, tracers are advected in cartesian coordinates to avoid deformation due to the spherical geometry. After advection, the composition field is updated on each grid node by perfoming trilinear interpolation with the surrounding tracers. For the diffusive part, a subgrid scale diffusion operation is performed similarly to Gerya and Yuen, 2003 [4]. This method was implemented in the code PARODY (E. Dormy, J. Aubert) and parallelised using MPI and OpenMP, permitting to run simulations within reasonable time. Several advection tests were performed to check that this method does not produce numerical diffusion and will be shown at EPSC (fig. 1).

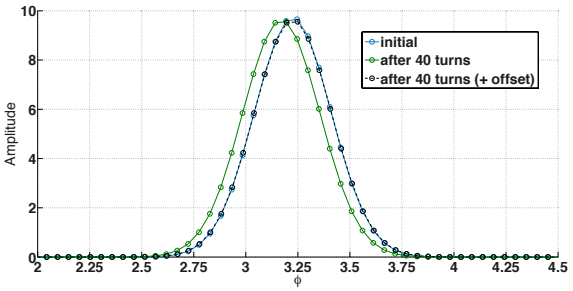

Figure 1: Profile of a gaussian equatorial patch before and after 40 turns of solid rotation around the equator. The shape has not changed confirming the absence of numerical diffusion during advection.

3. Benchmarking

In order to validate this method, we ran simulations on the benchmark case 1 proposed by Christensen et al., 2001 [3]. In this test, tracers are used to solve the transport equation for temperature. Note that since the gain with tracers resides mainly in the ability to treat low-diffusive fields, not in some improvement of the code's precision, we only need the results to be in correct agreement with the benchmark's standard solution. Preliminary results are given in table 1 and show sufficient agreement even at low resolution. Runs at higher resolution will be shown at EPSC as well as results on the thermo-chemical benchmark proposed by Breuer et al., 2010 [2].

Table 1: Preliminary benchmark case 1 results at low resolution ($N_r = 90, N_\theta = 66, l_{max} = 44$), with (Parody^{tra}) and without tracers (Parody). See [3] for the terms definitions.

	Standard	Parody	Parody^{tra}
E_{kin}	30.733	30.964	30.271
E_{mag}	626.41	629.27	623.70
T	0.37338	0.3730	0.3729
u_ϕ	-7.6250	-7.3465	-7.2855
B_θ	-4.9289	-4.9934	-4.9499
ω	-3.1017	-3.1200	-2.9960

4. Applications

Our present focus is to implement thermodynamically consistently coupled boundary conditions for temperature and composition. In a first step, we intend to apply this new tool to non-magnetic thermo-chemical convection in the ocean of Ganymede. The results obtained will be presented at EPSC.

Acknowledgements

Numerical simulations were performed on three different platforms at IDRIS, Orsay, CCIPL, Nantes and PSMN, Lyon, France. We also thank Guy Moebs and Cerasela Iliana Calugaru for technical support. This work also benefited from helpful discussions with Hagay Amit and Philippe Cardin.

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