Polarimetric Neutron Magnetic Field Tomography - a non-Abelian ray transform

Naeem M. Desai *[1], William R.B. Lionheart[1], Morten Sales[2], Markus Strobl[3], and Søren Schmidt[2]

[1]School of Mathematics, University of Manchester, Manchester, UK
[2]DTU Physics, Technical University of Denmark, Lyngby, Denmark
[3]European Spallation Source ERIC, Lund, Sweden

Keywords: Three-dimensional magnetic field imaging, reconstruction algorithm, neutrons, applications

Summary: We consider the problem to determine a magnetic field from Polarimetric Neutron Magnetic Field Tomography (PNMFT) data. The linearised problem about a zero magnetic field reduces to a plane Radon transform and a Modified Newton Kantarovich type algorithm is suggested for the numerical solution of the nonlinear problem. A range for which the Modified Newton Kantarovich type algorithm works is deduced.

1. INTRODUCTION

At the close of the last century, it was proposed to use the neutron magnetic moment for real space tomographic imaging investigations of magnetic structures. Simulations have been presented in [4] of a novel method called Neutron Magnetic Tomography.

However, a decade later an efficient, but restricted, experimental approach of Polarized Neutron Imaging was introduced to make two and three dimensional images of magnetic fields [2]. In contrast to initial proposals, the setup used does not measure the full spin rotation matrix but only a single diagonal element. Correspondingly, the method has been applied for strongly oriented fields and high symmetry cases providing significant *a priori* knowledge for analysis, e.g. through field modeling and simulation matching with data. In this manner magnetic fields of electromagnetic devices and electric currents, [2, 5, 6, 3, 7], but also quantum mechanical effects in superconductors [1, 2, 6] could be studied successfully.

A potential important application is the study of three dimensional magnetic configurations, in particular magnetic domain structures. The feasibility of such a methodology for this problem was proposed in [6]. Until now there have been no results on reconstruction algorithms or uniqueness of solution for this problem, although preliminary results are presented using the *ad hoc* method of scalar Radon transform inversion applied to polarized neutron data [2, 4]. We show that the problem can be formulated as a non-Abelian ray transform and that sufficiently smooth magnetic fields are uniquely determined by polarimetric neutron tomography data. We show that the linearized problem, for small magnetic fields, has a unique solution for less regular fields and propose an iterative reconstruction algorithm for the non-linear case, which we have implemented and tested on simulated data.

This lays the foundation for practical three dimensional magnetic field imaging that will facilitate imaging of magnetic domains in metal samples and the corresponding design of magnetic materials.

2. EXPERIMENTAL METHOD

Experimentally we assume a polarised neutron beam with the same wavelength and beam size of approximately 4×4 cm^2. In reality the beam has $\Delta\lambda/\lambda \approx 1\%$ that can be tuned by trading against the neutron flux. For the neutron wavelength range commonly used, polarisations well beyond 90% and close to 100% can be achieved with spin filters like super mirror devices as used for polarised neutron imaging [1, 2]. A neutron spin filter only transmits neutrons with spin parallel to the magnetization of the device. The analyser gives a unique determination of the final polarization state for rotations less than 180 degrees. A detailed explanation of the experimental setup can be seen in Figure 1(b) which is located at J-PARC, Tokai (Japan).

*e-mail: naeem.desai@manchester.ac.uk

3. RESULTS

A central slice of a solenoidal vector field (metal coil) was calculated using the Biot-Savart law on a 180×180 pixel grid. The initial spin data was simulated using 270 rays, where each ray is an ensemble of neutrons possessing a wavelength of 5 Å. These rays are fired for every angular increment (1 degree in this case) of the usual tomographic data acquisition process. The data was binned by a factor of 3 before 5% pseudo-random gaussian noise was added. Finally radon inversion was performed thrice for each component of the magnetic field, where the grid for reconstruction does not evenly divide the grid used for simulation, i.e. 67×67. The relative errors are $22\%, 17\%, 9\%$ and 10% for the magnetic field strength, $|B(x(t))|$ and components $B_1(x(t)), B_2(x(t))$ and $B_3(x(t))$ respectively. Results can be seen in Figure 1(a) where the comparison between standard radon inversion and the new iterative procedure can be clearly seen. It took 25 iterations to converge and the maximum a neutron precesses throughout this specific magnetic field is 88°.

References

[1] Martin Dawson, Ingo Manke, Nikolay Kardjilov, Andre Hilger, Markus Strobl, and John Banhart. Imaging with polarized neutrons. *New Journal of Physics*, 11(4):043013, 2009.

[2] Nikolay Kardjilov, Ingo Manke, Markus Strobl, Andre Hilger, Wolfgang Treimer, Michael Meissner, Thomas Krist, and John Banhart. Three-dimensional imaging of magnetic fields with polarized neutrons. *Nat Phys*, 4:339–403, 10 2008.

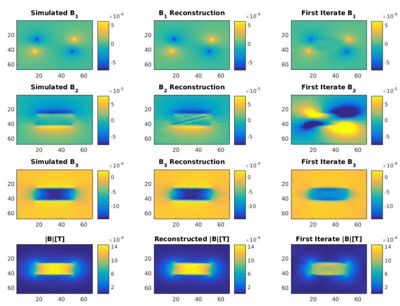
[3] I. Manke, N. Kardjilov, M. Strobl, A. Hilger, and J. Banhart. Investigation of the skin effect in the bulk of electrical conductors with spin-polarized neutron radiography. *Journal of Applied Physics*, 104, 076109 (2008). DOI : 10.1063/1.2992516.

[4] M. Hochhold, H. Leeb and G. Badurek. Tensorial Neutron Tomography : a first approach. *Journal of Magnetism and Magnetic Materials*, 157/158(1996) 575-576.

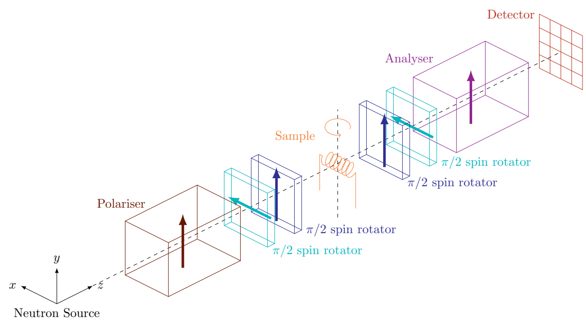
[5] T. Shinohara, K. Sakai, M. Ohi, T. Kai, M. Harada, K. Oikawa, F. Maekawa, J. Suzuki, T. Oku, S. Takata, K. Aizawa, M. Arai, and Y. Kiyanagi. Quantitative magnetic field imaging by polarized pulsed neutrons at J-PARC. *Nuclear Instruments and Methods A*, 651(1) : 121-125 (2011).

[6] M. Strobl, N. Kardjilov, A. Hilger, E. Jericha, G. Badurek and I. Manke. Imaging with polarized Neutrons. *Physica B*, 404 (2009) 26112614.

[7] A. Tremsin, N. Kardjilov, M. Strobl et al. Imaging of dynamic magnetic fields with spin-polarized neutron beams. *New Journal of Physics*, 17 043047 (2015). DOI : 10.1088/1367-2630/17/4/043047.



(a)



(b)

Figure 1: (a) Comparison between iterative procedure (middle column) and radon inversion (right column) results. Original magnetic field is the left column. (b) Instrumental set-up. The neutrons are polarised in the y-direction by the polariser, and the two $\pi/2$ spin rotators downstream can be used to rotate the neutron spins to the x- or z-directions. The sample is rotated along a vertical axis for different tomographic projections, and the two following $\pi/2$ spin rotators choose the direction of analysis before the analyser, which transmits neutrons with spins along y. At the end, the signal is recorded by a position sensitive detector.