



Climate change in a Point-Over-Threshold model: an example on ocean-wave-storm hazard in NE Spain

R. Tolosana-Delgado (1), M.I. Ortego (2), J.J. Egozcue (2), and A. Sánchez-Arcilla (1)

(1) Universitat Politècnica de Catalunya, Laboratori d'Enginyeria Marítima, Barcelona, Spain (raimon.tolosana@upc.edu; agustin.arcilla@upc.edu), (2) Universitat Politècnica de Catalunya, Departament de Matemàtica Aplicada III, Barcelona, Spain (ma.isabel.ortego@upc.edu; juan.jose.egozcue@upc.edu)

Climatic change is a problem of general concern. When dealing with hazardous events such as wind-storms, heavy rainfall or ocean-wave storms this concern is even more serious. Climate change might imply an increase of human and material losses, and it is worth devoting efforts to detect it. Hazard assessment of such events is often carried out with a *point-over-threshold* (POT) model. Time-occurrence of events is assumed to be Poisson distributed, and the magnitude of each event is modelled as an arbitrary random variable, which upper tail is described by a Generalized Pareto Distribution (GPD). Independence between this magnitude and occurrence in time is assumed, as well as independence from event to event.

The GPD models excesses over a threshold. If X is the magnitude of an event and x_0 a value of the support of X , the excess over the threshold x_0 is $Y = X - x_0$, conditioned to $X > x_0$. Therefore, the support of Y is (a segment of) the positive real line. The GPD model has a scale and a shape parameter. The scale parameter of the distribution is $\beta > 0$. The shape parameter, ξ , is real-valued, and it defines three different sub-families of distributions. GPD distributions with $\xi < 0$ have limited support ($y_{sup} = -\beta/\xi$). For values $\xi > 0$, distributions have infinite heavy tails ($y_{sup} = +\infty$), and for $\xi = 0$ we obtain the exponential distribution, which has an infinite support but a well-behaved tail. The GPD distribution function is

$$F_Y(y|\beta, \xi) = 1 - \left(1 + \frac{\xi}{\beta}y\right)^{-\frac{1}{\xi}}, \quad 0 \leq y < y_{sup} \quad \xi \neq 0,$$

with exponential limit form $F_Y(y|\beta, \xi = 0) = 1 - \exp\left(-\frac{y}{\beta}\right)$ for $\xi = 0$, where $0 \leq y < +\infty$.

Scarcity of data arises as an additional difficulty, as hazardous events are fortunately rare. A Bayesian approach seems thus quite a needed step to estimate the GPD parameters in a way that their uncertainty is adequately propagated to the hazard parameters, such as return periods. This bayesian perspective allows us to transparently include necessary conditions on the parameters for our particular phenomena. For instance, in our case study, we may be sure that there is a maximal height related to physical limitations (sea depth, fetch distance, water density, etc.). Thus, we choose as a priori statement that $\xi < 0$.

On the other hand, the selection of proper scales for the description of phenomena is an important issue as well. A handful of phenomena are better described by a relative scale (e.g. positive data where the null value is unattainable) and are thus suitably treated in a logarithmic scale. This has been already used successfully for daily rainfall data and ocean-wave-height.

How to assess impact of climate change on hazardous events? In a climate change scenario, we can consider the model for description of the variable as stable, while its parameters may be taken as a function of time. Thus, magnitudes are taken in a log-scale. Excesses over a threshold are modeled by a GPD with a limited maximum value ($\xi < 0$). The parameters of this model are considered as a function of time. In order to deal with this parameter change, a new parameterisation of the GPD distribution is suggested:

$$\mu = \ln(-\xi/\beta) ; \quad v = \ln(-\xi \cdot \beta),$$

where μ is a new location parameter, related to the upper limit of the distribution. With this reparameterization,

several hypothesis about parameter changes can be made: abrupt change in a point of time, change as a (lineal, logistic, ...) function of time, etc. For hazardous phenomena with a physical upper limit, the parsimonious choice is to consider a lineal change on v with time, whilst μ remains constant. Then, the climate change is assessed by the change on $v(t) = \nu_0 + t \cdot \Delta\nu$, with competing models:

$$M_0 : \Delta\nu = 0 \quad \text{vs.} \quad M_1 : \Delta\nu \neq 0$$

These issues are illustrated using a set of 18 years of significant-ocean-wave-height data measured in a buoy in front of the Ebro delta. A Bayesian joint estimation of parameters is carried out. Posterior and predictive distributions are available for the trend and no trend hypothesis. Results show that no significant change has been observed in the last 18 years.