The Inversion of Low-Temperature Thermochronometry to Extract Spatially Varying Exhumation Rates

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We present a formal inverse procedure to extract exhumation rates from spatially distributed low temperature thermochronometric data, such as apatite fission track, zircon fission track and (U-Th)/He. Our method is based on a Gaussian linear inversion approach in which we set up a linear problem relating exhumation rate to thermochronometric age with rates being parameterized as variable in both space and time. The basis of our linear forward model is the fact that the depth to the “closure isotherm” can be described as the integral of exhumation rate, \( e \), from the cooling age to the present day. This permits a set of ages to be described as a linear system of equations related to exhumation rate. The second assumption is that exhumation rates vary smoothly in space, and so can be described through a spatial correlation function, in which correlation decreases with increased separation distance. We discretize the exhumation history over a set of timesteps, but require that exhumation be spatially correlated over each timestep. Therefore the entire exhumation history of an orogen is described in time intervals, but is free to vary in space. We use an a priori estimate of the model parameters, in order to invert this linear system with a maximum-likelihood estimator. Inversion of all available thermochronometric ages thus provides an estimate of exhumation rate through all space and time. An estimate of the resolving power of the data is also obtained by computing the a posteriori variance of the parameters and comparing it directly to the a priori variance. The resolution of the data is a function of the spatial distribution, but also the elevation distribution. In particular, we note that with this method, ages co-located in space, but at different elevations automatically resolve an exhumation rate over their age range, equivalent to fitting a line to the age-elevation relationship.

To verify the model, a section of typical alpine topography was subject to a synthetic, spatially varying exhumation history, and a resulting suite of thermochronometric ages calculated. Through the use of this dataset we illustrate the ability of the algorithm to resolve spatially varying exhumation rates. We also investigate the number of ages required to retrieve the prescribed exhumation function by varying the number of synthetic samples used for the inversion, and the number of thermochronometric systems included. We also add random noise to the data to test the sensitivity of the model to natural variance. We thereby explore the dependency of the a posteriori model with respect to the a priori model, and the affects of dampening the solution. As there exists a trade off between resolving changes in exhumation rate through time, and minimizing the a posteriori variance, the choice of a time step length is investigated. Also discussed are preliminary inversion results for areas where a dense coverage of thermochronologic data are available, including the Alps, Corsica, and the Apennines.