European Conerence on Severe Storms 2019,
4–8 November 2019, Kraków, Poland, ECSS2019-219-1

Insights into Tornadogenesis from Integrals of the Vorticity Equation and from Angular-Momentum Advection

Robert Davies-Jones

Emeritus, National Severe Storms Laboratory

An axisymmetric numerical model in a closed domain (Davies-Jones 2008) illustrates the important role of downward angular momentum advection in tornadogenesis. The boundary conditions are no slip on the tangential velocity component and no shear stress on the radial and vertical components. The model uses precipitation drag to upset a balanced Beltrami initial state that resembles a central mid-level mesocyclone with a compensating outer downdraft. Without precipitation the flow pattern is perpetual and the flow amplitude decays slowly. The release of a moderate amount of precipitation through the top boundary has a drastic effect on the Stokes streamfunction ψ and angular momentum M fields. The contours of ψ and M coincide initially, and $\psi = M = 0$ on the boundaries throughout the simulation. The M contours serve as the vortex lines for the radial-vertical vorticity. Due to only minor diffusion, M is nearly conserved following trajectories.

A recent animation reveals the following tornadogenesis mechanism. The drag exerted by the precipitation streamer initiates tornadogenesis by upsetting the initially balanced midlevel mesocyclone. Precipitation drag intensifies the downdraft, which now lowers angular momentum. Outflow from the downdraft flows towards the axis, transporting M with it. This low-level air with moderately strong M is then drawn into the updraft. Owing to upward M advection, the updraft rotates faster, pressure falls, and the vortex aloft becomes more cyclostrophic. Because of extra radial mass influx, the Stokes streamfunction ψ changes in the corner region. But contrary to a dynamic-pipe effect, ψ changes little above the corner region. Consequently, the corner streamlines turn from sloping inward with height to vertical. The no-slip condition on tangential velocity now becomes very important. In the boundary layer, friction reduces the centrifugal force and the flow becomes sub-cyclostrophic with the unbalanced inward pressure-gradient force driving parcels even closer to the axis. The associated spin-up produces an intense tornado that breaks down into the wider weaker cyclone aloft. Clearly the tornado is a result of downward, inward *M* transport, and frictional interaction between the vortex and the ground. The simulation reproduces observed features such as a clear slot and anticyclonic vorticity in the tornado vicinity, and confinement of rising air near the ground to the tornado's high-speed axial jet.

We can use angular momentum as observed by single Doppler radar as a tool for tornado warning and tornadogenesis diagnosis. Circulation around a circle is 2π times the average angular momentum about the center of the parcels on the circle. From the mean Doppler velocity field we can calculate the circulation around and areal contraction rate of circles with radii 1 to 3 km concentric with vortices. Since Doppler radar only observes one velocity component, the Doppler circulation around and areal contraction rates of circles in surfaces of constant elevation angle are doubled to estimate the actual values. Averaging around a circle instead of around radar grid cells is advantageous because it removes spurious quadrupole patterns in the observed circulation field. Computations for the 24 May 1973 Union City, Oklahoma tornadic vortex signature (TVS) indicate that aloft the initial mesocyclone had totally contracted into the mature tornado within a broad region of constant convergence. Double the observed circulation aloft, 10^5 m^2/s, agreed well with the photogrammetrically observed circulation at 200 m height. The tornado was modeled as a convergent potential vortex and a virtual radar was used to compute its signatures at different ranges and azimuths. Circulation declines by less than 20% for ranges up to 90 times the circle radius and is relatively range insensitive compared to other measures of TVS strength (i.e., rotational velocity and shear).

Now consider the following vorticity processes that are important for tornadogenesis, but excluded from the numerical model because of its axisymmetry:

a. Tilting of environmental vorticity. Such tilting is understood tacitly to be the origin of the model's pre-existing updraft rotation.
b. The river-bend process. In the model azimuthal vorticity cannot be reoriented.
c. Likewise, buoyancy generated vorticity cannot be tilted.

Integrals of the Lagrangian equation for **w** (specific volume α times vorticity vector) provide theoretical perspectives into the roles of these processes in tornadogenesis. Setting the baroclinic and frictional torques to zero results in the homogeneous or barotropic **w** equation. The barotropic **w** of a given parcel P is obtained using the method of Dahl et al. (2014). The position vector $\mathbf{x}(\tau)$ of P at time τ is $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ where **i**, **j**, and **k** are unit eastward, northward, and upward vectors. At the initial time τ_0, $\mathbf{x}(\tau_0) = X\mathbf{i} + Y\mathbf{j} + Z\mathbf{k}$ where (X, Y, Z) are the Lagrangian coordinates of P. In a horizontally homogeneous pre-storm environment, the environmental vorticity is horizontal and a function of original parcel height Z. Furthermore, surfaces of constant Z ('Z-surfaces') are initially level material surfaces. Henceforth assume that the wind and the vorticity components are storm-relative.

Centered on P, we set up a tiny fluid stencil consisting of P and six neighboring parcels labeled E, W, N, S, A and B that initially are a distance Δ east, west, north, south, above, and below P, respectively, with Lagrangian coordinates $(X \pm dX, Y, Z)$, $(X, Y \pm dY, Z)$, and $(X, Y, Z \pm dZ)$, where $dX = dY = dZ = \Delta$. These points are the midpoints of the faces of a material cube. By the chain rule, the differential of position vector at time τ is $d\mathbf{x}(\tau) = \mathbf{e}_1(\tau)dX + \mathbf{e}_2(\tau)dY + \mathbf{e}_3(\tau)dZ$ where $\mathbf{e}_1(\tau) = \partial\mathbf{x}/\partial X$, $\mathbf{e}_2(\tau) = \partial\mathbf{x}/\partial Y$, $\mathbf{e}_3(\tau) = \partial\mathbf{x}/\partial Z$. Initially the $\mathbf{e}_i(\tau)$ (i = 1, 2, 3) are unit vectors equal to **i**, **j**, **k**, respectively, attached to P. As P moves through the storm, these material line elements behave like 'elastic strings' that stretch and turn with the flow. These 'string vectors' (formally known as the covariant basis vectors) are frozen in the fluid and propagate a parcel's barotropic vorticity through time by accounting for the "frozen field" effect. After the initial time, the material volume becomes a parallelepiped with the same mass as the initial cube. Mass conservation thus constrains the scalar triple product of the string vectors (i.e., material volume) divided by the specific volume to be invariant following the motion. Any linear combination of the string vectors is a solution of the barotropic **w** equation. The parcel's barotropic **w** is the particular linear combination that satisfies its initial condition (i.e., its environmental **w**). Because barotropic **w** is frozen in the fluid and the initial vertical vorticity is zero, the barotropic vorticity normal to each *Z*-surface is permanently zero. We may rotate the basis vectors in each *Z*-surface so that in the new basis ($\mathbf{e}'_1$, $\mathbf{e}'_2$, $\mathbf{e}_3$), $\mathbf{e}'_1$ is initially streamwise to the environmental wind and $\mathbf{e}'_2$ is initially 90° to the left of $\mathbf{e}'_1$ (i.e., transverse or crosswise to the environmental wind). The (contravariant) components of the barotropic vorticity in terms of the rotated covariant basis vectors are constant as they are the environmental streamwise and crosswise vorticity. Within the *Z*-surfaces the flow reorients and stretches or diminishes $\mathbf{e}'_1$ and $\mathbf{e}'_2$.

The integral of the Lagrangian baroclinic **w** equation with zero initial condition is

$$\frac{\mathbf{w}_{\mathrm{BC}}(\tau)}{\alpha(\tau_0)} = \mathbf{e}_1(\tau)\int_{\tau_0}^{\tau}\frac{\partial[b(\sigma),z(\sigma)]}{\partial(Y,Z)}d\sigma + \mathbf{e}_2(\tau)\int_{\tau_0}^{\tau}\frac{\partial[b(\sigma),z(\sigma)]}{\partial(Z,X)}d\sigma + \mathbf{e}_3(\tau)\int_{\tau_0}^{\tau}\frac{\partial[b(\sigma),z(\sigma)]}{\partial(X,Y)}d\sigma.$$

where z is parcel height and $b\nabla z$ (= $b\mathbf{k}$) is the buoyancy force. Thus we can compute $\mathbf{w}_{\mathrm{BC}}$ from the time integrals of *b*-*z* material solenoids at stencil points, and the current strings. Note that the contravariant components of $\mathbf{w}_{\mathrm{BC}}$ accumulate over time and the timing of the baroclinic vorticity generation within the interval $[\tau_0, \tau]$ is immaterial if the definite integrals remain the same. The solution of the Lagrangian frictional **w** equation, not given, has a similar form, but with three times as many terms.

We assume hereafter that the storm reaches a steady state so that the parcels follow the streamlines. The steady-state wind $\mathbf{v}(\tau)$ is simply equal to $q(\tau_0)\ \mathbf{e}'_1(\tau)$ where $q(\tau)$ is the wind speed so that e'_1 is *everywhere* streamwise in the 3D sense instead of being streamwise only in the environment. We now introduce orthonormal basis vectors **t**, **n**, **b** in the streamwise, transverse, and binormal (normal to *Z*-surface) directions. In this system $\mathbf{v}(\tau) = q(\tau)\ \mathbf{t}(\tau)$. The transverse vorticity has two terms. Term 1 is the transverse vorticity owing to the environmental crosswise vorticity being stretched or shrunk in proportion to the ratio of the current streamline spacing at the parcel P to the spacing in the environment. Term 2 is the transverse baroclinic vorticity accumulated by P over time. As P travels in anti-streamwise (streamwise) buoyancy gradients, it obtains positive (negative) transverse vorticity.

The expression for streamwise vorticity consists of terms A, B, C. Term A is streamwise vorticity imported from the environment and subsequently stretched along streamlines. Term B is accumulation of baroclinic streamwise vorticity, which is generated in positive transverse buoyancy gradients. Term C is streamwise vorticity created by turning of upstream positive transverse barotropic and baroclinic vorticity through the river-bend process. It is equal to the transverse vorticity component times cot ϕ where ϕ is the angle between $\mathbf{e}'_2$ and $\mathbf{e}'_1$ (or **t**). This process occurs in cyclonically curved flow lacking vorticity in the binormal direction. Positive transverse vorticity associates with windspeed increasing with height. As the flow bends, the faster (slower) fluid above (below) moves outward (inward) due to excess centrifugal force (pressure-gradient force), resulting in turning of $\mathbf{e}'_2$ towards the streamwise direction and streamwise vorticity.

Term A explains why abundant environmental streamwise vorticity in the storm inflow close to the ground favors tornadic supercells. This 3D streamwise vorticity flows directly into the base of storm updraft unmodified apart from streamwise stretching, thus establishing mesocyclonic rotation and low pressure in the updraft at low altitudes. The associated vortex suction can lift quite negatively buoyant air that may be underneath the updraft. Term B also plays a role in updraft rotation. Parcels in the "streamwise vorticity current" along the forward-flank downdraft boundary acquire baroclinic streamwise vorticity owing to being in a leftward transverse buoyancy gradient. Rotation is locally enhanced where these parcels enter the updraft.

Vorticity very close to the ground arises principally as follows. Parcels flowing into a downdraft are in an anti-streamwise buoyancy gradient and acquire positive transverse baroclinic vorticity through term 2. These subsiding parcels are turned to the left in the outer mesocyclone by the mesocyclone's inward pressure-gradient force and obtain 3D streamwise vorticity through term C, the river-bend mechanism. As the parcels exit the left side of the rear-flank downdraft near the ground and accelerate towards the updraft, their 3D streamwise vorticity amplifies owing to streamwise stretching associated with streamline confluence. Because mass is conserved, the *Z*-surfaces pack closer together. Beneath the rotating updraft vortex suction lifts the cool parcels. Upward tilting and vertical stretching of their 3D streamwise vorticity can sustain a tornado.