



## Multifractal 3D spatial-scale analysis of soil variables using wavelets

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In this work, we study 3D soil samples as acquired with CT (Computed Tomography) and processed in order to reconstruct a 16-bit 3D imagery from the sequence of axial views. As shown in previous work done by some of the authors [1], if we extract 3 sub-volumes of 256x256 pixels, the corresponding images seem to exhibit a multifractal behaviour, which can be further analysed using classical multifractal methodology or through a wavelet-based approach in order to quantify the soil pore distribution (both spatial and scale-dependent analysis is carried out).

Through 3D wavelet analysis of the entire volume (instead of processing each sub-volume separately), new results and physical interpretations could be derived but first, there is a need for establishing a wavelet-based methodology dealing with the increase of complexity resulting from two to three dimensions).

To build wavelets in higher dimension we first use tensor products of one-dimensional constructions resulting in separable filters which give preferential treatment to the coordinate axis. Although the construction of such a wavelet is conceptually simple, the methodology applied in order to compute multifractal parameters is fully revised and presented along with some insights into the interpretation of the derived results.

The separable approach only allows for rectangular divisions of the frequency spectrum. Often, other symmetry axes than the coordinate ones and nonrectangular divisions of the frequency spectrum offer better results. The construction of wavelets over these other support or lattices as known in multidimensional signal processing, relies on the lifting scheme [3]. The extension of our methodology to this second generation wavelets provides other advantages which are preliminary explored in our work.

Both methodologies are illustrated using natural 3D volumes and well-known fractal structures as previous work before analyzing soil samples.

[1] J.A. Piñuela, A. Alvarez, D. Andina, R.J. Heck and A.M. Tarquis, "Quantifying a soil pore distribution from 3D images: multifractal spectrum through wavelet approach", Submitted to Geoderma. April 2008.

[2] J. Kovacević and W. Sweldens, "Wavelet families of increasing order in arbitrary dimension" IEEE Trans on Image Processing, Volume 9, Issue 3, Mar 2000 Page(s):480 – 496.

[3] W.Sweldens, "The lifting scheme: A construction of second generation wavelets", SIAM J. Math. Anal. 29(2). 1997