



"Fast" evolution of wind wave spectra at the spectral cut-off

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Experimental wind wave spectra have nearly universal shape in the frequency domain: there is a well pronounced spectral peak, with a power-like “slope” for higher frequencies and quite steep “cut-off” for the frequencies below that of the peak. There exists a number of parameterizations of such spectra with a few tuning parameters. The spectra are self-similar with the peak slowly downshifting. On the theoretical side it was found that the main features of such spectra could be explained by employing self-similar solutions of the kinetic (alias, Hasselmann and spectral transfer) equation. A fairly good agreement with the experimental data was found for the slope part of the spectrum and the dynamics of the spectral peak, while the “front” of the spectrum corresponding to the cut-off received very little attention.

Within the framework of the kinetic equation the spectra of any random water wave field evolve on the ε^{-4} timescale, where ε is the characteristic wave steepness and wave frequencies are presumed to be $O(1)$. Recently we have found that the kinetic equation works well only in a certain vicinity of its equilibrium solutions. Strong perturbations evolve towards equilibrium much faster than the kinetic equation predicts, on the dynamic ε^{-2} timescale rather than the ε^{-4} kinetic timescale. The perturbations were assumed to be due to external factors, such as, e.g. rapid change of wind, nonuniform currents, etc. The fundamental underlying cause of fast spectral evolution is the dependence of time derivatives of the spectra on the history of the field evolution.

Here we show that even in the generic situation of perfectly uniform and steady conditions wave spectra develop fronts. At these fronts the classic kinetic equation is not applicable and wave spectra evolve on the fast “dynamic” scale. The analogy with the gas dynamics could be helpful: the kinetic equation describes macroscopic evolution of the field, playing the same role as the Navier-Stokes equations. In gas dynamics wide classes of initial conditions lead to the formation of shocks, and the inner structure of shocks is often not described by the Navier-Stokes equations. In wave kinetics the fronts are analogues of the shocks, and inapplicability of the kinetic equation is similar to the failure of the Navier-Stokes equations in gas dynamics.

We demonstrate the fast evolution at the front by direct numerical simulation within the framework of the Zakharov equation. The numerical method includes building of a non-regular grid of several thousand harmonics, coupled by $O(10^7)$ approximately resonant interactions. The spectral evolution at the front is also investigated analytically and numerically within the framework of the generalized kinetic equation able to describe fast evolution.

The phenomenon of fast nonlinear evolution at the fronts is not confined to wind waves, it occurs for all wave fields evolving due quartet nonlinear interactions wherever there is inverse cascade resulting in formation of fronts.

References

- [1] S.Y. Annenkov and V.I. Shrira. “Fast” nonlinear evolution in wave turbulence. *Phys. Rev. Letters* **102**, 2009.