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Characterizing dynamics and hyperbolicity with covariant Lyapunov vectors

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Structural stability (i.e. topological invariance under small parameters perturbations) is a key property in dynamical systems theory, since it guarantees the robustness of the predictions obtained through a given dynamical model. Structurally stable systems, in turn, are essentially hyperbolic ones, that is, their tangent space splits into well separated expanding and contracting subspaces (respectively called unstable and stable manifolds).

In this talk we present the first reliable numerical method to verify hyperbolicity (and eventually to quantify the magnitude of its violations) in high dimensional systems. This is achieved by computing covariant Lyapunov vectors, that is the complete set of stable and unstable tangent space directions associated - at each point in phase space - with exponential growth rates (the Lyapunov exponents). Covariant Lyapunov vectors differ from other phase space and tangent space vectors commonly used in the literature (such as Bred vectors, singular vectors or Gram-Schmidt orthogonal vectors), since the former are covariant with respect to the dynamics and locally coincide with the stable and unstable manifolds. They can be efficiently computed in high dimensional systems through a forward-backward algorithm we have recently introduced.

Once these directions have been computed over a large number of phase space points which sample the entire strange attractor, the separation between the stable and unstable manifolds can be easily checked computing the distribution of the minimal angle between arbitrary linear combinations of contracting and expanding covariant Lyapunov vectors. A distribution that is not strictly bounded away from zero indicates the existence of homoclinic tangencies, phase phase points where the stable and unstable subspaces are not well separated and hyperbolicity is violated. By further analyzing the dimension of such tangencies and their frequency in phase space, one can also gather information on the magnitude of such violations.

Since the covariant Lyapunov vectors allow one to identify all the unstable directions along a given trajectory, they may also prove as a useful too in practical applications, such as instability control algorithms in atmospheric modeling.