



## **Random dynamical systems, SPDEs and applications**

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Modelling of physical processes, including climate modelling, is usually based on nonlinear PDE systems. Randomization of these models, leading to Stochastic Partial Differential Equations (SPDE) is necessary or interesting for a number of reasons. The most basic one is the lack of knowledge of the precise value of certain parameters, due to impossibility of measurements or unpredictability. In this case we usually have to do with PDE with random parameters, theoretically simple but numerically hard if simple Monte Carlo simulations are ruled out by the computational complexity of the system. A similar origin of stochasticity is the crude description of certain degrees of freedom by means of stochastic processes. But also a more pragmatic reason is the fact that SPDE may have properties which are more difficult to identify in deterministic systems, properties related to long time statistics and also to the transient regime. The talk will review a number of recent ideas and progresses on these topics. In addition, a related viewpoint will be summarized, based on the concept of random dynamical system (RDS), of which SPDE are one of the main examples. RDS take a more geometric viewpoint, as classical dynamical systems. Among other features, RDS investigate the simultaneous action of a noise realization on different initial conditions, capturing properties like synchronization. As an example, the RDS approach to a simple pitchfork bifurcation in a differential equation will be shown to change the bifurcation into a one point random attractor. Finally, a few remarks will be given about the challenging problem of the zero-noise limit.