



Stochastic processes and scaling anisotropy: Generalized Scale Invariance (GSI) and Operator Scaling fields.

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Generalized Scale Invariance (GSI) can be understood as a radical paradigm shift with respect to the principle of local isotropy argued by Kolmogorov to derive the scaling of atmospheric turbulence: the basic principle becomes to posit first scaling, then to study the remaining symmetries.

As we argued it sometime ago, this requires a generalized notion of scale in order to deal both with scale symmetries and strong anisotropies, e.g. time vs. space, vertical vs. horizontal. Surprisingly, the Fractal Geometry preserved the classical Euclidean distance to define the topology and the metric of the physical space, in particular to define the Hausdorff measure and dimension. There are on the contrary at least three main reasons to question the relevance of this metric for most of the geophysical processes. The first is that it is meaningless to assume isotropy between time and space because they are incommensurable. The second is that gravity induces in a general manner a strong anisotropy in space. Thirdly, there is no mathematical compelling reason to use the Euclidean distance to define the Hausdorff measure and dimension: a (generalized) notion of scales is readily obtained with the help of a one parameter anisotropic contraction semi-group.

We recall how GSI is rather indispensable to understand, analyse and model geophysical phenomena ranging from atmospheric turbulence to porous media: atmospheric motions and hydraulic conductivity are $23/9 D$ rather than being quasi-2D or quasi-3D.

We discuss the relationship between linear GSI and the notion of Operator Scaling fields. We emphasize that the latter is rather restricted to simple scaling, whereas the former was introduced and developed in the framework of multifractals, which rather correspond to multiple scaling than simple scaling. Finally, we discuss the case of nonlinear GSI, which is obviously much more general.