



## **A New Consistent Parameter-Choice Method for Tikhonov-Phillips Regularization Method, achieved and applied in Downward Continuation**

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Downward Continuation is a famous Inverse Problem in Geodesy and Inverse Problems are also famous in Mathematics. As often as not, these problems suffer from an illness called "Ill-Poseness" and the illness is often diagnosed as "Instability" and the ill due the instability has treatments called Regularizations and among the treatments there is a well-defined prescription referred to Dr. Tikhonov and Dr. Phillips (Actually at back of the prescription there is only a traditional treatment known as Least Squares). They prescribed an optimization based on a considered item called "Regularization Parameter". Fortunately, now the prescription is available but regrettably, it is scrawled to some extent and a question is remained for the patients "What is a Consistent Value for Regularization Parameter?" Also unfortunately, a unique consistent parameter-choice method has not been especially found for each regularization method yet and there are only some general parameter-choice methods which are founded on their assumptions separately and it is too difficult to distinguish which of them is better for which regularization method?!

The authors based on their experiences obtained a belief that the available Choice-Parameter Methods are general and can not lead us to a Consistent Parameter for every Regularization Method; they also believe every Regularization Method has a particular Choice-Parameter Method rising in behind the regularization assumptions. The authors in this paper do not claim that they innovated a New Consistent Parameter-Choice Method but they claim that they found it, maybe by chance!!!, during the curing an Ill-Posed problem achieved in Downward Continuation for Geoid Computation. But at first, it seems better to reveal briefly how they could meet it. Obviously, there are some statistical assumptions behind Tikhonov method which can recast it as Statistical Inversion Method. The main assumptions are Normality and Homoscedasticity of Observations which were also so fruitful for the authors to design a statistic and subsequently to carry out a Statistical Hypothesis Test. The test executes a Confidence Interval for the Problem Unknowns. Basically, the Confidence Interval has led us to its satisfying Parameter.

At first the paper begins with reviewing some statistical assumptions behind Tikhonov-Phillips Regularization Method; this motivates us to design a Statistic and to fulfill a Statistical Hypothesis Test for unknowns, so its result is a Confidence Interval for unknowns. Then the paper proposes some logics to choose Consistent Parameter which satisfies aforementioned Confidence Interval and finally the paper ends with applying the Consistent Choice-Parameter Method in Downward Continuation based on inversion of Abel-Poisson Integral Equation, for Geoid Computation in geographical region of Iran, as case study.