



Ensemble sizes required for particle filters in high dimensions

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Particle filters are ensemble-based state-estimation techniques that in principle approximate the correct Bayesian analysis step for general non-Gaussian probability distributions. We investigate the ensemble size necessary for the particle filter as the state dimension increases.

For the simplest particle-filter algorithm, in which the prior distribution (i.e. the distribution of the state at the present time, conditioned on previous observations) is used as the proposal distribution, simulations and asymptotic analysis (following Bengtsson, Bickel and collaborators) demonstrate that the required ensemble size scales exponentially with a certain measure of the problem size. When each component of the state vector is independent, Gaussian, and of unit variance and the observations are of each state component separately with independent, Gaussian errors, the required ensemble size scales exponentially with the state dimension and simulations show that at least 10^{11} members when applied to a 200-dimensional state. In more general cases, the asymptotic theory reveals that the ensemble size must scale exponentially with the variance of the observation log likelihood rather than with the state dimension per se.

A proposal density sufficiently close to the correct posterior would alleviate these difficulties, although there is no theoretical guidance for what sufficiently close means as the state dimension increases. Simulations indicate that the “optimal” proposal density of Doucet, which minimizes the variance of the particle weights after resampling, also suffers from an exponential increase of the necessary ensemble size.