



Nonlinear development of inertial instability in a barotropic shear

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Inertial instability is investigated with idealized numerical simulations in order to describe its nonlinear development and saturation. In order to focus on fundamental mechanisms, we consider a simple barotropic shear on the f -plane. For a velocity profile given by $U(y) = \tanh y$, analytical solutions and growth rates are obtained.

A major difficulty for the numerical simulations without explicit diffusion is that linear theory predicts that the instability will grow at the smallest available vertical scales. Hence, simulations have been run at different resolutions and with different levels of diffusion, and the linear development in the simulations is compared with the analytical solutions for validation. Satisfactory agreement is found: the growth rates are comparable, and the structure of the growing mode in the strongly diffusive simulations is nearly identical to the theoretical prediction.

The strongly diffusive simulations provide a simple scenario for the nonlinear development of the instability: as the mode becomes of finite-amplitude, it spreads horizontally. This leads to severe distortions of the initially vertical band of unstable fluid, producing strong vertical gradients. These are then dissipated by the vertical diffusion, and the fluid thus returns to a barotropic state, but with the shear spread out over a wider region, such that the final state has become marginally stable.

In fact, it is shown that the final state can be predicted based on the conservation of momentum. Remarkably, the barotropic component of the final state is always close to the simple theoretical prediction, regardless of the resolution and diffusion.

On the other hand, resolution and diffusion strongly affect the details of the nonlinear development and of the baroclinic component of the final state. When diffusion is sufficiently small and resolution sufficiently high, significant small-scale features are produced by the instability: free gravity waves which propagate away from the unstable region, trapped subinertial waves and layers of alternating weaker and stronger stratification. The subinertial waves and the stratification staircase are signatures which persist in the anticyclonic shear after it has become marginally stable.