



## Using the minimum description length principle for global reconstruction of dynamic systems from noisy time series

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We propose a new approach to determining embedding dimension when reconstructing dynamic systems from noisy time series.

The available methods of the reconstructing typically include two main steps: (1) reconstruction of the system's phase variables and (2) construction of a model reproducing behavior of the system in the corresponding region of phase space. Reconstruction of phase variables is accomplished, for example, by the method of delay coordinates [1] in the space of dimension referred to as embedding dimension. The embedding dimension should preferably be chosen to be minimum possible. In the absence of additional information about the system, the principal technique for determining embedding dimension is the false nearest neighbor method [2] that is easily realized. Unfortunately, this method is inefficient when the observed time series contains a pronounced noise component [3], thus making it inapplicable for reconstruction of natural systems. The basic feature of the second step – construction of a model from noisy time series – is its incorrectness. Namely, there always exist an infinite number of solutions approximating the observed data with preset accuracy. It is intuitively clear that for the great majority of applications the model will be the better the simpler it is. The authors of [4] proposed to use description length as a measure of such simplicity. The principle of minimum description length implies that the model corresponding to the least description length is the best. As was demonstrated in [5], this provides an effective tool for choosing technical parameters of the model, including the optimal number of such parameters.

In the current work we use the principle of minimum description length (MDL) for determining embedding dimension. For this we take the universal model in the form of an artificial neural network that includes embedding dimension as a parameter. The specific feature of using neural networks is the need to introduce physically based prior restrictions on network parameters; hence, we generalize the definition of the description length for this case. Besides, we introduce the MDL invariance requirement relative to arbitrary smooth transformations of model parameters. This requirement enables, in particular, finding an explicit expression for MDL, thus simplifying the use of the MDL principle significantly.

The new approach as we show is appreciably less sensitive to the level and origin of noise, which makes it also a useful tool for determining embedding dimension when constructing stochastic models.

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