



Testing for scale-invariance in extreme events, with application to earthquake occurrence

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We address the generic problem of testing for scale-invariance in extreme events, i.e. are the biggest events in a population simply a scaled model of those of smaller size, or are they in some way different? Are large earthquakes for example ‘characteristic’, do they ‘know’ how big they will be before the event nucleates, or is the size of the event determined only in the avalanche-like process of rupture? In either case what are the implications for estimates of time-dependent seismic hazard? One way of testing for departures from scale invariance is to examine the frequency-size statistics, commonly used as a bench mark in a number of applications in Earth and Environmental sciences. Using frequency data however introduces a number of problems in data analysis. The inevitably small number of data points for extreme events and more generally the non-Gaussian statistical properties strongly affect the validity of prior assumptions about the nature of uncertainties in the data. The simple use of traditional least squares (still common in the literature) introduces an inherent bias to the best fit result.

We show first that the sampled frequency in finite real and synthetic data sets (the latter based on the Epidemic-Type Aftershock Sequence model) converge to a central limit only very slowly due to temporal correlations in the data. A specific correction for temporal correlations enables an estimate of convergence properties to be mapped non-linearly on to a Gaussian one. Uncertainties closely follow a Poisson distribution of errors across the whole range of seismic moment for typical catalogue sizes. In this sense the confidence limits are scale-invariant. A systematic sample bias effect due to counting whole numbers in a finite catalogue makes a ‘characteristic’-looking type extreme event distribution a likely outcome of an underlying scale-invariant probability distribution. This highlights the tendency of ‘eyeball’ fits unconsciously (but wrongly in this case) to assume Gaussian errors. We develop methods to correct for these effects, and show that the current best fit maximum likelihood regression model for the global frequency-moment distribution in the digital era is a power law, i.e. mega-earthquakes continue to follow the Gutenberg–Richter trend of smaller earthquakes with no (as yet) observable cut-off or characteristic extreme event. The results may also have implications for the interpretation of other time-limited geophysical time series that exhibit power-law scaling.