



Flexible space-time process for seismic data

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Introduction

Point processes are well studied objects in probability theory and a powerful tool in statistics for modeling and analyzing the distribution of real phenomena, such as the seismic one. Point processes can be specified mathematically in several ways, for instance, by considering the joint distributions of the counts of points in arbitrary sets or defining a complete intensity function. The conditional intensity function is a function of the point history and it is itself a stochastic process depending on the past up to time t .

In this paper some techniques to estimate the intensity function of space-time point processes are developed, by following semi-parametric approaches and diagnostic methods to assess their goodness of fit.

In particular, because of its particularly adaptive properties to anomalous behavior in data, in this paper a non-parametric estimation approach is used to interpret dependence features of seismic activity of a given area of observation; to justify the estimation approach a diagnostic method for space-time point processes is also revised.

Flexible modeling and diagnostics for point processes

The definition of effective stochastic models to adequately describe the seismic activity of a fixed area is of great interest in seismology, since a reliable description of earthquakes occurrence might suggest useful ideas on the mechanism of a such complex phenomena.

A number of statistical models have been proposed for representing the intensity function of earthquakes. The simpler models assume that earthquakes occur in space and time according to a stationary point process, such that conditional rate becomes a constant. In seismology, however, the stationarity hypothesis might be acceptable only with respect to time, because epicenters usually display a substantial degree of spatial heterogeneity and clustering. Description of seismic events often requires the definition of more complex models than stationary Poisson process and the relaxation of the assumption of statistical independence of earthquakes. Therefore, second-order properties of point processes may have a relevant role in the study and the comprehension of seismic process and its realization. Indeed, when aggregation is present, it is useful to introduce some generalizations of the simple Poisson process, such as self-exciting point processes, to model events that are clustered together, and self-correcting processes when regularities are observed, e.g. the strain-release model (Daley and Vere-Jones, 2003). A widely used model is ETAS model (Ogata, 1988), that is a self-exciting point process, describing earthquakes activity, in a given region during a period of time, through a branching structure. Also in this field, the parametric models estimation suffers by many drawbacks, often related to the definition of a reliable mathematical model from the geophysical theory and to the sensitivity of statistical estimates to the composition of the sample, that is the space-time region under study.

Many of the disadvantages of the parametric modeling can be avoided by making use of nonparametric techniques, such as kernel density methods (Silverman, 1986). Therefore a flexible model, that is useful in presence of several data for which a not immediately obvious discrimination between principal and secondary events is not reliable, estimated by nonparametric method is proposed. In particular, a three dimensional Gaussian kernel estimator is used (Adelfio and Ogata, 2008), developing a method of estimation with predictive features, taking into account the dependence on the past history of a point process. For this purpose variation of the likelihood function to measure the capability of the observations until a fixed time to give information on the next observation is considered (Adelfio and Chiodi, 2008a).

To assess the goodness of fit of a given model diagnostic methods are necessary (Adelfio and Chiodi, 2008b). Therefore, a new method for point processes is here considered (Adelfio and Schoenberg, 2008); it is based on the interpretation of second-order statistics (such as R/S, correlation integral, spectral density) weighted by a quantity proportional to the inverse of the conditional intensity function. Such transformed statistics are useful to test the fit of space-time point processes when features like self-similarity, long-range dependence and fractal dimension have to be taken into account for a deeper comprehension of the observed phenomena.

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