Geophysical Research Abstracts,
Vol. 11, EGU2009-3458-1, 2009
EGU General Assembly 2009

To development of analytical theory of rotational motion of the Moon

Yu.V. Barkin (1), J.M. Ferrandiz (2), and J.F. Navarro (2)

(1) Sternberg Astronomical Institute, department of celestial mechanics and gravimetry, Moscow, Russian Federation (barkin@inbox.ru, 07-495-9328841), (2) Alicante University, department of applied mathematics, Spain

Resume. In the work the analytical theory of forced librations of the Moon considered as a celestial body with a liquid core and rigid non-spherical mantle is developed. For the basic variables: Andoyer, Poincare and Eulerian angles, and also for various dynamic characteristics of the Moon the tables for amplitudes, periods and phases of perturbations of the first order have been constructed. Resonant periods of free librations have been estimated. The influence of a liquid core results in decreasing of the period of free librations in longitude approximately on 0.316 day, and in change of the period of free pole wobble of the Moon on 25.8 days. In the first approximation the liquid core does not render influence on the value of Cassini's inclination and on the period of precession of the angular momentum vector. However it causes an additional "quasi-diurnal" librations with period about 27.165 days. In comparison with model of rigid non-spherical of the Moon the presence of a liquid core should result in increase of amplitudes of the Moon librations in longitude on 0.06 %.

1 Development of analytical theory of rotational motion of the Moon with liquid core and rigid mantle. The work has been realized in following stages. 1. Canonical equations of rotation of the Moon with liquid core and elastic mantle in Andoyer and Poincare variables have been constructed. Developments of second harmonic of force function of the Moon in pointed variables have been obtained for accurate trigonometric presentation of perturbations of the Moon orbital motion. 2. Two approaches (two methods) of construction of analytical theory have been developed. These approaches use different principles for eliminating of singularities for axial rotation of the Moon. One is based on direct application of Andoyer variables by changing of notations of moments of inertia [1]. Second is based on application of Poincare elements. For comparison both approaches are developed. 3. The main equation for determination of Cassini's inclination and its solution has been obtained in the case of accurate orbit of the Moon. An dynamical explanation of Cassini's laws has been done for model of the Moon with liquid core [2]. 4. Compact formulae for perturbations of the first (and second) order have been constructed for general used variables and for different kinematical and dynamical characteristics of the Moon (23 variables and characteristics: Andoyer-Poincare variables, classical variables, components of angular velocity and angular momentums of the Moon and its core). 5. Analytical formulae for 4 periods of free librations of the Moon have been constructed: for librations in longitude, in pole wobble, for free precession, and "quasi-diurnal" librations, caused by the liquid core. 6. The dynamical effects in the Moon rotation, caused by secular orbital perturbations of the Earth and Sun, have been studied.

2 Structure perturbations of the first order and their tabulation. For example, perturbations (periodic and of mixed type) in inclination $\rho$and in node h of angular momentum of the Moon are determined by formulae: $\rho = \rho_0 + \sum\limits_{\nu} \rho_\nu^{(1)} \cos\theta_{\rm v}$, $h = \pi + \sum\limits_{\|\nu\|} h_\nu^{(1)} \sin\theta_\nu$. Here $\rho_0 = 1^0 33' 50"$ is the Cassini's inclination of the Moon; $\rho_\nu^{(1)}$, $h_\nu^{(1)}$are constant coefficients; $\theta_{\rm v} = v_1 l_M + v_2 l_S + v_3 F + v_4 D$, $\nu = (v_1, v_2, v_3, v_4)^{\rm T}$are combinations of known classical arguments of the Moon orbital theory; v_1, v_2, v_3 and v_4 are integer.

3 Influence of the liquid core and its ellipticity ε on amplitudes of the Moon forced and free librations. An influence of the liquid core and its ellipticity is determined by positive correction to amplitudes of librations for model of the rigid Moon. If the amplitudes of librations of rigid Moon we note as 1, so the corresponding amplitudes of librations of the Moon with the liquid core will be characterized by parameter $1+L$, where correction for liquid core is determined by formula $L = C_c \left(1 - \varepsilon^2\right)/C \approx C_c/C = 0.5996 \cdot 10^{-3}$, where $C$and $C_c$is the polar moments of inertia of the Moon and its core;$\varepsilon = \left(a^2 - b^2\right)/\left(a^2 + b^2\right) \approx (a-b)/a$ is an ellipticity of equatorial ellipse of core cavity with semi-axes a and b. So all amplitudes of librations in longitude due to the liquid core are increased on 0.06%. A small effect of ellipticity has more smaller order. Here as example we present formula for perturbations of the first order of the Moon in longitude:

$$\lambda^{(1)} = 6n_0^2 \frac{1+L}{I} C_{22} \times$$

$$\times \sum_{\|\nu\|>0} \sum_{\nu_5} (-1)^{\nu_5} \frac{D^{(1)}_{\nu_1.\nu_2.\nu_3+2.\nu_4.\nu_5}(\rho_0) - D^{(-1)}_{\nu_1.\nu_2.\nu_3-2.\nu_4.\nu_5}(\rho_0)}{(v_1 n_M + v_2 n_S + v_3 n_F + v_4 n_D)^2} \sin(v_1 l_M + v_2 l_S + v_3 F + v_4 D)$$

$I = C/(mr^2)$ is the dimensionless moment of inertia of the Moon (m and $r$are it's the mass and mean radius). Kinoshita's inclination functions $D^{(\pm 1)}_{\nu_1.\nu_2.\nu_3.\nu_4.\nu_5}(\rho_0)$ are determined by known formulae through the value of Cassini's angle$\rho = 1^0 33' 50''$. $v_1 n_M + v_2 n_S + v_3 n_F + v_4 n_D = \dot{\theta}_{v_1,v_2,v_3,v_4}$ are derivatives with respect to the time of corresponding linear combinations of classical arguments of lunar orbit theory; n_M, n_S, n_F and n_D are velocities of changes of these arguments; C_{22} is the selenopotential coefficient; $n_0^2 = f m_\oplus / a^3$, a is an unperturbed value of semi-axis major of lunar orbit, $f$is a gravitational constant. The perturbations of the first order for others variables and considered dynamical characteristics have the structure similar to the formula for $\tilde{\lambda}^{(1)}$. In given table 1 we present amplitudes of forced librations in longitude of intermediate Andoyer plane $\lambda_{\nu_1,\nu_2,\nu_3,\nu_4}$ (in arc seconds) and perturbations of angular velocity of the Moon axial rotation $\omega_{\nu_1,\nu_2,\nu_3,\nu_4}$ (in units$10^{-4} n_F$). $T_{\nu_1,\nu_2,\nu_3,\nu_4}$are periods of corresponding perturbations.

Table 1. Main perturbations in the Moon librations in longitude.

ν_1	ν_2	ν_3	ν_4	$T_{\nu_1,\nu_2,\nu_3,\nu_4}$	$\lambda_{\nu_1,\nu_2,\nu_3,\nu_4}$
0	**1**	**0**	**0**	**365.26**	**81"02**
1	**0**	**0**	**0**	**27.555**	**-15"65**
1	**-1**	**0**	**-1**	**-3232.9**	**9"85**
2	**0**	**0**	**-2**	**205.89**	**9"69**
1	**0**	**0**	**-2**	**31.81**	**4"15**
1	**0**	**0**	**-1**	**411.78**	**-2"98**
2	**0**	**-2**	**0**	**-1095.2**	**-1"86**
2	**-1**	**0**	**-2**	**471.89**	**0"74**
0	**0**	**0**	**2**	**14.77**	**-0"61**

The results of tabulations of amplitudes of perturbations in the Moon rotation give good agreement with earlier constructed theories for its rigid model. Barkin's work partially was financially accepted by Spanish grants, Japanese-Russian grant N-07-02-91212 and by RFBR grant N 08-02-00367.

References

[1] Barkin, Yu. (1987) An Analytical Theory of the Lunar Rotational Motion. In: Figure and Dynamics of the Earth, Moon and Planets/ Proceedings of the Int. Symp. (Prague, Czechoslovakia, Sept. 15-20, 1986)/ Monogr. Ser. of UGTK, Prague. pp. 657-677.

[2] Ferrandiz, J., Barkin, Yu. (2003) New approach to development of Moon rotation theory. Procced. of Inter. Conf. "Astrometry, Geodynamics and Solar System Dynamics". Journees 2003 (Sept. 22-25, 2003, St. Peters., Russia). IPA RAS, 199-200.