When can we assume Lagrangian stationarity?

N. Suciu (1), C. Vamoş (2), H. Vereecken (3), K. Sabelfeld (4,5), and P. Knabner (1)

(1) Mathematics Department, Friedrich Alexander University Erlangen-Nuremberg, Germany (suciu@am.uni-erlangen.de, knabner@am.uni-erlangen.de), (2) Tiberiu Popoviciu Institute of Numerical Analysis, Romanian Academy, Cluj-Napoca (cvamos@ictp.acad.ro), (3) Institute of Agrosphere, Research Center Jülich, Germany (h.vereecken@fz-juelich.de), (4) Weierstrass Institute for Applied Analysis and Stochastics, Berlin, Germany (sabelfel@wias-berlin.de), (5) Institute of Computational Mathematics and Mathematical Geophysics, Siberian Branch of Russian Academy of Sciences, Novosibirsk, Russia (karl@osmf.sscc.ru)

The statistical stationarity in space of the Lagrangian velocity and the translation-invariance of the ensemble average Green function are often key assumptions in stochastic modelling of transport in natural porous media. We show that the two statements are equivalent and hold true when the Eulerian velocity field is statistically homogeneous and the transport equations admit unique solutions. Sufficiently smooth velocity samples that guarantee unique solutions are ensured by the existence of the derivatives of the correlation functions at the origin, as for instance in case of Gaussian correlations. These sufficient conditions can hardly be relaxed and are very close to the necessary conditions. Random velocity fields with exponential correlations obviously do not fulfill the sufficient sample-smoothness conditions. Numerical approximations of such fields also show their lack of smoothness and simulations of transport indicate dependence on initial conditions of the ensemble average Lagrangian velocity and Green function, i.e. statistical inhomogeneity. Therefore, the question whether Lagrangian stationarity can be assumed in case of exponentially correlated fields remains a challenging issue.