Geophysical Research Abstracts,
Vol. 11, EGU2009-3719-5, 2009
EGU General Assembly 2009

Plane Mercury librations

Yu.V. Barkin (1) and J.M. Ferrandiz (2)

(1) Sternberg Astronomical Institute, department of celestial mechanics and gravimetry, Moscow, Russian Federation (barkin@inbox.ru, 07-495-9328841), (2) Alicante University, department of applied mathematics, Spain (jm.ferrandiz@ua.es)

Introduction. In 1988 I. Kholin [1] has developed a precision method of determination of parameters of rotation of planets on complex radar-tracking observations on two radio telescopes making base and definitely carried on surface of the Earth. His American colleagues for the period approximately in 4 with small year have executed a series of radar-tracking measurements on a method and I. Kholin's program [2] and have obtained for the specified period 21 values of angular velocity of rotation of this planet [3]. With the help of numerical integration of the equations of rotary motion on the found values they managed to determine with high accuracy the basic dynamic parameter in the theory of Mercury librations $(B-A)/C_m = (2.03 \pm 0.12) \cdot 10^{-4}$ and the corresponding to it the value of amplitude of the basic librations$35"8 \pm 2"1$. These results have served as convincing arguments for the benefit of the Peale's assumption, that a core of Mercury is liquid, or in partially molten [4]. Authors also managed to obtain for the first time parameters of resonant librations in a longitude which opening from radar observations was predicted earlier [5]. Its amplitude makes about 300", the period is equal approximately to 12 years.

In the paper [6] parameters of the perturbed rotational motion have been determined with the help of the analytical theory and with formal using of results of mentioned work [3] on determination of 21 values of angular velocity of Mercury. In result the estimations of amplitudes of forced librations of first five harmonics with the periods: 87.97 d, 43.99 d, 29.33 d, 21.99 d and 17.59 d have been obtained. The appropriate amplitudes make values:34"05±1"27, $3"59 \pm 0"13$, $0"354 \pm 0"013$, $0"072 \pm 0"003$ and $0"016 \pm 0"001$. The amplitude and the period of free librations of Mercury in a longitude are determined: $290"9 \pm 67"0$ and 12.37 ± 0.23 yr, consequently. The phase of this variation has made$284^0 1 \pm 14^0 2$.

In the paper we construct the similar theory of Mercury librations in longitude by using three characteristics of Mercury rotation determined in the paper [3]. Two from these parameters are values of angle of librations in longitude and angular velocity in moment of passage of perihelion of Mercury orbit on 17 April 2002: $(\delta g)_0 = 0^0 07 \pm 0^0 01$, $(\delta\omega/\omega)_0 = (2.10 \pm 0.06)/\omega$ ars/d. Third parameter determined in [3] is a dynamical coefficient: $K = (B-A)/(4C_m) = (5.08 \pm 0.30) \cdot 10^{-5}$. $B > A$ are principal moment of inertia, corresponding to equatorial axes of inertia; C_m is a polar moment of inertia of the mantle of Mercury.

1 Analytical theory of plane Mercury librations. This theory describes forced and free librations of Mercury in longitude in the frame of plane problem about resonant librations of Mercury considered or as non-spherical rigid body, or as system of rigid non-spherical mantle and liquid ellipsoidal core. Saving the main terms for the perturbations of angle of librations δg and angular velocity $\delta\omega$ in both mentioned cases we will have formulae [6]:

$$\delta g = K\left(E_1 \sin M + E_2 \sin 2M + E_3 \sin 3M + E_4 \sin 4M + E_5 \sin 5M\right) + K_0 \sin\left(E\sqrt{K}M - \varphi\right) \text{ (A)}$$

$$\frac{\delta\omega}{\omega} = 2K\left(E_1 \cos M + 2E_2 \cos 2M + 3E_3 \cos 3M + 4E_4 \cos 4M + 5E_5 \cos 5M\right)/3+$$

$$+2E\sqrt{K}K_0 \cos\left(E\sqrt{K}M - \varphi\right)/3.$$

In the case of Mercury model as a rigid non-spherical body its dynamic parameter$K = C_{22}/I$, where $I = C/\left(mr^2\right)$ is the dimensionless moment of inertia of Mercury (m and $r$are its mass and mean radius; C is the polar moment of inertia). For a Mercury model with a liquid ellipsoidal core this parameter can be reduced to the following [6]: $K = (C_{22}/I) \cdot (C/C_m)$ (here $C_m$is a polar monent of inertia of the mantle). Coefficients$E$ and E_i have concrete numerical values [6].

First five trigonometrically terms in (A) describe (with the accepted accuracy) the forced librations of Mercury in longitude, and last term describes a free long-periodic librations in longitude. Here $M = n(t - t_0)$ is the mean anomaly of Mercury. $n = 4.092339765\ {}^{0}\!\big/$day is the mean angular velocity of orbital motion of Mercury. $T = 2\pi/n = 87.969257$ d is the orbital period of Mercury. t_0 is a moment of passing of pericenter of Mercury orbit, as which we shall accept the date $t_0 = 52381.480$ MJD, following [3]. K_0 and φ is the amplitude and phase of free librations in longitude. These characteristics can be determined only on the base of observations. $E\sqrt{K} \cdot n$ and $T_g = T/\left(E\sqrt{K}\right)$ is the frequency and period of free librations.

The computing problem consist in determination of three parameters K, K_0 and phase φ on the base of 21 values of angular velocity from the work [3]. From these parameters only the parameter K_0 is included into expressions (A) linearly, concerning other parameters the problem is nonlinear. In this connection the nonlinear method of least squares has been applied. The initial expressions has been linearized and estimations of parameters have been calculated interracially. In result the analytical theory of plane librations has been constructed. Parameters of these theory have been described in Introduction.

2 Semi-analytical theory of Mercury librations. This theory is constructed on the base of three parameters determined in [3]. So using the above mentioned parameters$(\delta g)_0 = 0^0 07 \pm 0^0 01$, $(\delta\omega/\omega)_0 = (2.10 \pm 0.06)/\omega$ ars/d finally we obtain analytical formulas of plane librations of Mercury in following form:

$$(\delta\omega/\omega) \cdot 10^5 = (11.58 \pm 0.68) \cos M - (2.44 \pm 0.14) \cos 2M - (0.361 \pm 0.021) \cos 3M-$$

$$- (0.098 \pm 0.006) \cos 4M - (0.027 \pm 0.002) \cos 5M+$$

$$+ (1.771 \pm 0.498) \cos \left[(1.996 \pm 0.030) \cdot 10^{-2} M + 66^0 7 \pm 24^0 4\right],$$

$$\delta g = (35\text{"}81 \pm 2\text{"}12) \sin M - (3\text{"}78 \pm 0\text{"}22) \sin 2M - (0\text{"}372 \pm 0\text{"}022) \sin 3M+$$

$$- (0\text{"}076 \pm 0\text{"}005) \sin 4M - (0\text{"}017 \pm 0\text{"}001) \cos 5M+$$

$$+ (274\text{"} \pm 77\text{"}) \cos \left[(1.946 \pm 0.030) \cdot 10^{-2} M + 66^0 7 \pm 24^0 4\right].$$

In the report the dynamic effects in Mercury rotation caused by influence of the third and higher harmonics of force function, by influence of planetary orbital perturbations in motion of Mercury, by tidal deformations in diurnal rotation and in motion of poles of Mercury are discussed.

The work was financially accepted by RFBR grant N 08-02-00367.

References

[1] Kholin I.V. (1988) Space-time coherence of a signal diffusely scattered by an arbitrarily moving surface in the case of monochromatic sounding. Radiofizika (ISSN 0021-3462), vol. 31, no. 5, p. 515-518. In Russian.

[2] Kholin, I. V. (2004) Long-range coherence of the radar field scattered by a rotating mercury. Solar System Research, Volume 38, Issue 6, pp.449-454.

[3] Margot J.L., Peale S.J., Jurgens R.F., Slade M.A., Holin I.V. (2007) Large longitude libration of Mercury reveals a molten core. Science 316, 710; DOI: 10.1126/science.1140514.

[4] Peale, S. J. (1976) Does Mercury have a molten core? Nature, vol. 262, Aug. 26, 1976, p. 765, 766.

[5] Barkin, Yu. V.; Ferrandiz, J. M. (2005) Mercury: Libration, Gravitational Field and Its Variations. 36th Annual Lunar and Planetary Science Conference, March 14-18, 2005, in League City, Texas, abstract no. 1075.

[6] Barkin Yu.V., Ferrandiz J.M., Zotov L.V. (2008) Semi-empirical model of free and forced Mercury librations in longitude on the data of ground radar-tracking observations. Abstract Book (CD) of European Planetary Science Congress (Munster, Germany, 21 – 26 September 2008), Vol.3, EPSC 2008-A-00140.