



Proper orthogonal decomposition of velocity gradient fields in a simulated stratified turbulent wake: analysis of vorticity and internal waves

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The characterization of three-dimensional space and time-dependent coherent structures and internal waves in stratified environment is one of the most challenging tasks in geophysical fluid dynamics. Proper orthogonal decomposition (POD) is applied to 2-D slices of vorticity and horizontal divergence obtained from 3-D DNS of a stratified turbulent wake of a towed sphere at $Re=5 \times 10^3$ and $Fr=4$.

The numerical method employed solves the incompressible Navier-Stokes equations under the Boussinesq approximation. The temporal discretization consists of three fractional steps: an explicit advancement of the nonlinear terms, an implicit solution of the Poisson equation for the pseudo-pressure (which enforces incompressibility), and an implicit solution of the Helmholtz equation for the viscous terms (where boundary conditions are imposed). The computational domain is assumed to be periodic in the horizontal direction and non-periodic in the vertical direction. The 2-D slices are sampled along the stream-depth (Oxz), span-depth (Oyz) and stream-span planes (Oxy) for 231 times during the interval, $Nt \in [12, 35]$ (N is the stratification frequency). During this interval, internal wave radiation from the wake is most pronounced and the vorticity field in the wake undergoes distinct structural transitions.

POD was chosen amongst the available statistical tools due to its advantage in characterization of simulated and experimentally measured velocity gradient fields. The computational procedure, applied to any random vector field, finds the most coherent feature from the given ensemble of field realizations. The decomposed empirical eigenfunctions could be referred to as “coherent structures”, since they are highly correlated in an average sense with the flow field. In our analysis, we follow the computationally efficient method of ‘snapshots’ to find the POD eigenfunctions of the ensemble of vorticity field realizations.

The results contains of the separate POD modes, along with the reconstructed vorticity and horizontal divergence fields based on the linear combination of the eigenfunctions. Similar to applications of POD to the characterization of coherent structures in turbulent boundary layers, characteristic geometrical features for each eigenmode of vorticity and horizontal divergence are deduced.

The results show that in the Oxz plane at the wake centerline the first, most energetic, modes of vorticity reveal a structure similar of the forward-inclined vertical shear layers typical of late-time stratified wakes. In Oxz planes, off-set from the wake centerline, the signature of internal waves in the form of forward-inclined coherent beams extending into the ambient becomes evident. The angle of inclination becomes progressively vertical with increasing POD mode. Lower POD modes on the Oyz planes show a layered structure in the wake core with coherent beams radiating out into the ambient at angles spanning 0 to 75 degrees. The POD analysis of horizontal divergence on the Oxz and Oyz planes reveals similar features with the results for the vorticity field. Two notable exceptions at lower modes are the less organized structure of the wake core and the predominance of beam-like structures in laterally offset Oxz planes. Furthermore, these differences are confirmed through the relative energy spectra distribution of the eigenmodes for the vorticity and the horizontal divergence. Qualitative comparison of the reconstructed low-order velocity gradient fields and the computed flow fields shows the relative contribution of the different mode combinations, to the various flow features such as internal waves and vorticity. It is shown that POD analysis has provided a statistical description of the geometrical features previously observed in instantaneous flow fields of stratified turbulent wake.