



## Modeling of seismic waves in layered media and the inversion of source parameters

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This paper is organized as follows. After a discussion of the differential equations for wave propagation in the horizontally stratified medium and of the initial and boundary conditions, we derive the displacements on the free surface of the layered medium for plane waves when a point source is located on the  $s$ —th imaginary boundary at the depth  $z_s$  (physical parameters of the layers  $s$  and  $(s + 1)$  are put to be identical). Then, the source will be represented as a single force of arbitrary orientation and a general moment tensor point source. Further, “a primary field” for a point source will be introduced. Recurrent method for the solution of the direct seismic problem is considered based on the matrix method of Thomson-Haskell. The tensor represents a superposition of three single couples without moment along the  $x$ ,  $y$ ,  $z$ -axes and three double couples in  $xy$ ,  $xz$ ,  $yz$ -planes. Further, we give the results for the field of displacements on the free surface. The far-field displacements are:

$$\begin{pmatrix} u_z^{(0)} \\ u_r^{(0)} \end{pmatrix} = \sum_{i=1}^3 \int_0^\infty k^2 I_i L^{-1} [M_i g_i] dk, \quad u_\varphi^{(0)} = \sum_{i=5}^6 \int_0^\infty k^2 J_i L^{-1} [M_i g_{i\varphi}] dk \quad (1)$$

$$I_1 = \begin{pmatrix} J_1 & 0 \\ 0 & J_0 \end{pmatrix}, \quad I_2 = \begin{pmatrix} J_0 & 0 \\ 0 & J_1 \end{pmatrix}, \quad I_3 = I_2.$$

$$\begin{aligned} \mathbf{g} &= \begin{pmatrix} g_{iz} \\ g_{ir} \end{pmatrix}, \quad J_5 = J_0, \quad J_6 = J_1. \text{ The near-field displacements are:} & \begin{pmatrix} u_r^{(0)} \\ u_\varphi^{(0)} \end{pmatrix} = \frac{1}{r} \cdot \left( \int_0^\infty k \cdot J_1(kr) \cdot L^{-1} \left[ \begin{pmatrix} M_1 \\ -M_5 \end{pmatrix} \cdot (g_{1r} + 2g_{5\varphi}) \right] dk + \right. \\ & \left. + \int_0^\infty \left( k J_0(kr) - \frac{2J_1(kr)}{r} \right) \cdot L^{-1} \cdot \left[ \begin{pmatrix} -M_4 \\ M_6 \end{pmatrix} \cdot (g_{3r} + 2g_{6\varphi}) \right] dk \right) \quad (2) \end{aligned}$$

$$u_z^{(0)} = \frac{1}{r} \cdot \int_0^\infty k J_1(kr) \cdot L^{-1} \cdot [M_4 \cdot g_{3z}] dk,$$

where

$$\begin{aligned} M_1 &= M_{xz} \cos \varphi + M_{yz} \sin \varphi, \\ M_2 &= M_{zz}, \\ M_3 &= \cos^2 \varphi \cdot M_{xx} + \sin^2 \varphi \cdot M_{yy} + \sin 2\varphi \cdot M_{xy}, \\ M_4 &= -\cos 2\varphi \cdot M_{xx} + \cos 2\varphi \cdot M_{yy} - 2 \sin 2\varphi \cdot M_{xy}, \\ M_5 &= M_{yz} \cos \varphi - M_{xz} \sin \varphi, \\ M_6 &= \sin 2\varphi \cdot M_{xx} - \sin 2\varphi \cdot M_{yy} - 2 \cos 2\varphi \cdot M_{xy} \end{aligned} \quad (3)$$

The results of this direct problem (1-3) we use in the inversion of source parameters. The inverse method relies on inverting for components of the moment tensor and a determination of an earthquake source-time function.