



## Geostatistical inversion of transient moment equations of groundwater flow

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We present a methodology for conditioning estimates of hydraulic heads and fluxes and their associated uncertainty on information about transmissivity,  $T$ , and hydraulic heads,  $h$ , collected within a randomly heterogeneous aquifer under transient conditions. Our approach is based on recursive finite-element approximations of exact nonlocal first and second conditional moment equations. We present a nonlinear geostatistical inverse algorithm for transient groundwater flow that allows estimating jointly the spatial variability of log-transmissivity,  $Y = \ln T$ , the underlying variogram and its parameters, and the variance-covariance of the estimates. Log-transmissivity is parameterized geostatistically based on measured values at discrete locations and unknown values at discrete "pilot points." While prior pilot point values are obtained by generalized kriging, posterior estimates at pilot points are obtained by history matching of transient mean flow against values of hydraulic head collected during a pumping test. Parameters are then projected onto a computational grid by kriging. Prior information on hydraulic properties is included in the optimization process via a suitable regularization term which is included in the objective function to be minimized. The weight of the regularization term, hydraulic and unknown variogram parameters are then estimated by maximum likelihood calibration. The main features of the methodology are explored by means of a synthetic example. As alternative flow models we consider (a) a second-order and (b) a lower-order closure of the mean transient flow equation and assess the ability of these models at capturing the parameters of the estimated log-transmissivity variogram. With the aid of formal model selection criteria we associate each mean flow model and different sets of tested variogram parameters with a weight, or posterior probability, representing their relative degrees of likelihood. Our findings suggest that the weight of the regularization term is best identified by adopting a complete second-order approximation of the mean flow model, while predictions of  $Y(\mathbf{x})$  and  $h(\mathbf{x}, t)$  only marginally benefit from a second-order correction. Analysis based on posterior model weights based on the *Kashyap* measure,  $KIC$ , sharply identify the second-order based mean flow model as the most reliable. A unique feature of the method is its capability of providing estimates of prediction errors of hydraulic heads and fluxes, which are calculated a posteriori, upon solving corresponding moment equations. Our example shows that conditioning transient flow predictions on information of both transmissivity and hydraulic heads in general brings about a notable reduction of predictive uncertainty.