



Determination of the gradient of curvature of the plumbelines of the normal gravity field and a initial study of its isocurvature lines

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The curvature k of a plumbline of the Earth's normal gravity field U passing through a point P is a function which contains the first and second order partial derivatives of the normal potential U (referring to a Cartesian system). To determine the gradient of curvature at P the third order partial derivatives of the normal potential are also needed. However the determination of these high order partial derivatives demands too many complicated and tedious calculations. Here we describe a method to determine the gradient of curvature without using the third order partial derivatives of U . As a first step we express the partial derivatives of normal potential U in a global Cartesian system (X, Y, Z) such that the Z -axis is the Earth's mean axis of rotation, the X -axis is the intersection of the equator's plane and the plane of the Greenwich meridian and the Y -axis makes the system right-handed. For the problem at hand, we first introduce a local Cartesian (x, y, h) system such that a) the x - axis is tangent to the parallel circle at $\phi = \phi_P$, b) the y - axis is tangent to the meridian $\lambda = \lambda_P$ and c) the h - axis is the vertical to the ellipsoid passing through the point P . Subsequently we introduce a local Cartesian system (x_1, y_1, h_1) whose center is the point P and the transformation equations are $x_1 = x$, $y_1 = y$, and $h_1 = h_P - h$. Now in the interior of a circle of radius δ (δ is less than a meter) which has as a center the point P and lies on the meridian plane of P we assume that the coordinates of the $gradU$ change linearly and the second order partial derivatives of U practically do not change. In the interior of the circle - we name it D - we construct a function $k_a = k_a(y_1, h_1)$ with the use of which we determine the curvature of a plumbline at a specific point in the set D . The function k_a is a quotient of polynomial functions and it is a good approximation of the function k in the set D . Hence it is easy to determine the $gradk_a$ in terms of the (x_1, y_1, h_1) coordinates in D and consequently at the point P . Finally using the coordinate transformations we express the $gradk_a$ in the global Cartesian system (X, Y, Z) .

The isocurvature lines are curves such that if k is the function which describes the curvature of the plumbelines then at each point it holds that $k(X, Y, Z) = k_o = const$. We prove that there are at least two isocurvature lines which pass through a point P , they are orthogonal to each other and both of them are plane curves. Next we prove that these two curves lie on a special surface which is the isocurvature surface passing through the point P and finally we prove that the isocurvature surfaces are surfaces of revolution. The study of these new geometrical entities may reveal more properties of the normal gravity field.