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Improved Estimation of Moment Scaling in Multifractal Processes

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Consider a stationary multifractal measure in the unit interval and denote by ε_n, $n \geq 0$, the average measure density in a generic sub-interval of length 2^{-n}. Multifractality implies that the non-diverging moments of ε_n satisfy $E[\varepsilon_n^q] \propto 2^{nK(q)}$ where $K(q)$ is a convex moment-scaling function. The problem we address is how to most accurately estimate $K(q)$ from observations at some resolution $2^{n_{dat}}$. A popular estimator of $K(q)$ is the slope of the least-square regression of $\log_2\langle\varepsilon_n^q\rangle$ against n for $n = n_1, ..., n_{dat}$, where $\langle\ \rangle$ denotes sample averaging and $n_1 \geq 0$ is a parameter. Not much is known about the performance of this estimator, except that above some moment order q' the estimator tends to be linear in q. Our objective is to determine the bias and variance of this and other estimators and suggest improvements.

For positive moments and in general for noiseless data, we consider extended estimators of the following type. Denote by $n_0 \geq 0$ a resolution level below n_{dat}. We partition the unit interval into 2^{n_0} sub-intervals of length 2^{-n_0} and view each sub-record as a shorter realization of the process. We apply the conventional regression estimator of $K(q)$ to each sub-record (under the constraint $n_1 \geq n_0$) and obtain the final estimator as the average of the sub-record estimators. The conventional estimator corresponds to $n_0 = 0$. Since the sub-record estimators are statistically independent, averaging them is an effective means to reduce the error variance. For given n_{dat}, the modified estimator depends on $(n_0,\ \Delta n_1)$, where $n_0 \geq 0$ and $\Delta n_1 = n_1 - n_0$ ranges from 0 to $n_{dat} - n_0 - 1$. From extensive Monte Carlo simulation, we have found that increasing n_0 increases the bias but reduces the error variance while increasing Δn_1 has beneficial effects on the bias. Since $n_0 + \Delta n_1$ cannot exceed $n_{dat} - 1$, there are tradeoffs between the two parameters and the choice that minimizes the RMS error is nontrivial. Conventional estimators (with $n_0 = 0$) tend to have lower bias but significantly higher variance than the optimal estimators.

Especially for the negative moments, a practical concern is the robustness of the estimator of $K(q)$ against noise and other data inaccuracies. To reduce the effect of these imperfections, one typically uses variants of the so-called wavelet-transform-modulus-maxima (WTMM) method. We show that these methods are generally biased and propose simple unbiased alternatives.

Finally we consider the case when $K(q; \underline{\theta})$ has known parametric form with unknown parameters $\underline{\theta}$. In this case it is typical to first estimate $K(q)$ nonparametrically as indicated above and then find $\underline{\theta}$ to best fit the nonparametric estimates. It is often feasible to use simulation to assess the bias of such estimators, either systematically as a function of the true value of $\underline{\theta}$ or, after a first estimate of $\underline{\theta}$ has been obtained, in the vicinity of that estimate. This allows one to correct for bias and select the parameters n_0 and Δn_1 to minimize the error variance. The minimum is typically attained for $(n_0 = n_{dat} - 1;\ \Delta n_1 = 0)$. Optimal bias-corrected estimators of $\underline{\theta}$ are much more accurate than conventional estimators.

We illustrate all suggested estimators for the case of lognormal multifractal measures.