



A delay differential model of ENSO variability: Extreme values and stability analysis

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We consider a delay differential equation (DDE) model for El-Niño Southern Oscillation (ENSO) variability [Ghil et al. (2008), *Nonlin. Proc. Geophys.*, 15, 417-433.] The model combines two key mechanisms that participate in ENSO dynamics: delayed negative feedback and seasonal forcing. Toy models of this type were shown to capture major features of the ENSO phenomenon [Jin et al., *Science* (1994); Tziperman et al., *Science* (1994)]; they provide a convenient paradigm for explaining interannual ENSO variability and shed new light on its dynamical properties. So far, though, DDE model studies of ENSO have been limited to linear stability analysis of steady-state solutions, which are not typical in forced systems, case studies of particular trajectories, or one-dimensional scenarios of transition to chaos, varying a single parameter while the others are kept fixed. In this work we take several steps toward a comprehensive analysis of DDE models relevant for ENSO phenomenology and illustrate the complexity of phase-parameter space structure for even such a simple model of climate dynamics.

We formulate an initial value problem for our model and prove the existence, uniqueness, and continuous dependence theorem. We then use this theoretical result to perform detailed numerical stability analyses of the model in the three-dimensional space of its physically relevant parameters: strength of seasonal forcing b , atmosphere-ocean coupling κ , and propagation period τ of oceanic waves across the Tropical Pacific. Two regimes of variability, stable and unstable, are reported; they are separated by a sharp neutral curve in the (b, τ) plane at constant κ . The detailed structure of the neutral curve becomes very irregular and possibly fractal, while individual trajectories within the unstable region become highly complex and possibly chaotic, as the atmosphere-ocean coupling κ increases. In the unstable regime, spontaneous transitions occur in the mean temperature (i.e., thermocline depth), period, and extreme annual values, for purely periodic, seasonal forcing. The model reproduces the Devils bleachers characterizing other ENSO models, such as nonlinear, coupled systems of partial differential equations; some of the features of this behavior have been documented in general circulation models, as well as in observations. We analyze the values of annual extremes and their location within an annual cycle and report the phase-locking phenomenon, which is connected to the occurrence of El-Niño events during the boreal (Northern Hemisphere) winter. We report existence of multiple solutions and study their basins of attraction in a space of initial conditions. We also present a model-based justification for the observed quasi-biennial oscillation in Tropical Pacific SSTs.

We expect similar behavior in much more detailed and realistic models, where it is harder to describe its causes as completely. The basic mechanisms used in our model (delayed feedback and forcing) may be relevant to other natural systems in which internal instabilities interact with external forcing and give rise to extreme events.